TORCH

The Operations Research Challenge

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University of Toronto
Operations Research Group
(CORS/INFORMS Toronto Student Chapter)

Instructions

- You have 2 hours and 30 minutes to write the first part of the competition. This booklet will be collected at 12:45PM.
- Please first write your team number on each page of this booklet. Note that failure to do so may result in your answers getting lost and your team receiving no marks for the corresponding questions.
- For each question, there is space in this booklet for writing your answer. Please write your final solutions in the indicated places, in the format explained in the questions. Answers stated on any other papers will NOT be marked.

1 The Piles of Pizza Delivery Service [150 marks]

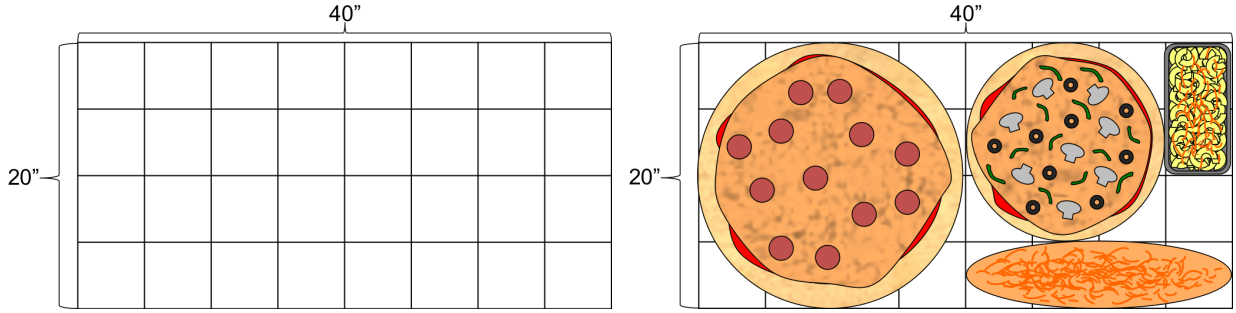
Toni is the owner of Piles of Pizza (PoP), a small high quality pizzeria located in Markham, just outside of Toronto. With its authentic take on some classic favourites, PoP has quickly become a popular choice for hosting small to medium sized events all over the Greater Toronto Area. Table 1 shows a list of PoP's most popular menu items, along with their respective cooking times, required oven spaces, and prices.

Item	Name (Shorthand)	Cooking Time	Required Oven Space	Price ($)
1	Large Pizza (L)	25 min	20"×20"	18
2	Small Pizza (S)	20 min	15"×15"	13
3	Garlic Bread (G)	15 min	5"×20"	10
4	Pasta (P)	10 min	5"×10"	7

Table 1: PoP's menu items with their cooking times, required oven spaces, and prices.

PoP's customers must place their orders at least 12 hours prior to their events. When the order is due, Toni's employees will drive to the customer's venue and will deliver the food. PoP guarantees that each customer will receive a $1 discount for every one minute an order arrives late (to a maximum of the total price of the order), and a free $10 voucher for any delivery that arrives cold. This means that a cold and late delivery costs Toni up to the price of the order plus $10.

In the past, all the orders have been delivered on time, so Toni has not needed to follow up on either of his guarantees. However, as PoP's region-wide reputation has expanded, so has its demand. Since Toni's kitchen is equipped with a single 40"×20" pizza oven, depicted in Figure 1, timely deliveries will become increasingly difficult to enforce.



(a) Empty oven. (b) All 4 items arranged in the oven.

Figure 1: PoP's 20"×40" oven. Each square is 5"×5".

Upon receiving tomorrow's list of lunch orders, shown in Table 2, Toni recognizes that he may have some late deliveries and calls an emergency staff meeting to ask for suggestions.

Order #	Items Ordered	Travel Time	Order Due (PM)
1	1×Small Pizza (S) 2×Large Pizza (L) 5×Pasta (P)	25 min	12:15
2	4×Garlic Bread (G) 5×Small Pizza (S)	10 min	12:30
3	2×Pasta (P) 3×Large Pizza (L) 2×Garlic Bread (G)	40 min	12:55
4	1×Large Pizza (L) 2×Small Pizza (S) 2×Garlic Bread (G) 2×Pasta (P)	5 min	1:30
5	3×Pasta (P) 2×Small Pizza (S) 3×Garlic Bread (G)	30 min	1:45

Table 2: PoP lunch orders for March 22nd, 2015 (tomorrow).

As the kitchen staff members, you have noticed that on a regular day, items are placed in the oven in the same sequence as they appear on the order list until no more item fits. Since you are familiar with Operations Research, you know that this is not the best way to use the oven space. Your suggestion is to improve the use of the oven space. Toni likes your idea and asks you to make an oven schedule that will maximize tomorrow's PoP profit. He also cautions you to remember the following rules:

- Food may stay in the oven longer than its designated cooking time, but never less (in which case it is considered "spoiled" and cannot be delivered).
- All food in a given batch must be placed in and removed from the oven at the same time. For example, if pasta and a small pizza are placed together in the oven, BOTH items must cook for 20 minutes, even though the pasta is ready after 10 minutes.
- Food has "gone cold" if it has been out of the oven for more than 40 minutes (not including the travel time). An order arrives cold if at least one of its food items has gone cold.
- Food cannot be stacked in the oven or divided into smaller pieces.
- Food preparation is done in advance, so you only need to consider the cooking time and the travel time.
- The store opens tomorrow at 11AM, i.e., the earliest time food can be put in the oven is 11AM.

The total possible profit from tomorrow's five orders is $432. However, if PoP uses the schedule given in Table 3, the total profit is $172, surprisingly smaller than the total possible profit.

Batch	**Time in the Oven**	**# of Items/Item/Order#**	**Order # Finished**	**Arrival Time**	**Due**	**Cold?**	**Profit**
1	11:00-11:25	1S1, 1L1	-				
2	11:25-11:50	1L1, 5P1, 1G2	1	12:15	12:15	N	$84
3	11:50-12:10	3G2, 2S2	-				
4	12:10-12:30	2S2	-				
5	12:30-12:55	1S2, 2P3, 1L3	2	1:05	12:30	Y	$105-$35-$10
6	12:55-1:20	2L3	-				
7	1:20-1:45	2G3, 1L4	3	2:25	12:55	Y	$88-$88-$10
8	1:45-2:05	2S4, 2G4, 2P4, 1P5	4	2:10	1:30	N	$78-$40
9	2:05-2:25	2P5, 2S5, 2G5	-				
10	2:25-2:40	1G5	5	3:10	1:45	N	$77-$77

Table 3: Current schedule for tomorrow's lunch orders.

Question

Improve the current schedule in Table 3.

Answer

The new schedule has _____ batches, for a total profit of $_____. Write your schedule in the table on the next page. Add more rows if necessary.

Batch	**Time in the Oven**	**# of Items/Item/Order#**	**Order # Finished**	**Arrival Time**	**Due**	**Cold?**	**Profit**

2 Periodic Maintenance of Electricity Networks [100 marks]

An electricity distributor provides electricity for nine different cities in Ontario including Toronto, Ottawa, Mississauga, Brampton, Hamilton, London, Markham, Vaughan, and Windsor. The network of each city is divided into two smaller networks. To ensure that customers in different cities do not experience frequent interruptions in service, each network must be periodically maintained. The maintenance service of each network includes transportation, equipment setup for inspection, treatment, and replacement.

The duration and frequency of a network's maintenance service, shown in Table 4, vary based on the network's size. For example, the maintenance service in Toronto's first sub-network (Toronto-1) takes 11 hours and should be done every 2 months. If Toronto-1 is maintained in February, then it must also be maintained in April, June, August, and so on.

Network	**Duration (hours)**	**Frequency (months)**	**Network**	**Duration (hours)**	**Frequency (months)**
Toronto-1	11	2	Hamilton-2	3	3
Toronto-2	9	2	London-1	3	3
Ottawa-1	5	2	London-2	13	6
Ottawa-2	4	2	Markham-1	11	6
Mississauga-1	4	2	Markham-2	10	6
Mississauga-2	2	2	Vaughan-1	5	6
Brampton-1	1	2	Vaughan-2	2	6
Brampton-2	4	3	Windsor-1	2	6
Hamilton-1	3	3	Windsor-2	1	6

Table 4: Duration and frequency of maintenance services in different networks.

Table 5 shows the current schedule of the maintenance services in a 6-month planning horizon. The total workload in each month equals the sum of the durations of the maintenance services scheduled in that month. The distributor must have enough workers and equipment on hand to handle all maintenance work. The distributor therefore plans to budget for the busiest month, i.e., the month with the highest number of total maintenance hours.

Based on the schedule in Table 5, February has the highest workload of 43, determining the budget for performing all maintenance work.

Service	**January**	**February**	**March**	**April**	**May**	**June**
Toronto-1		11		11		11
Toronto-2		9		9		9
Ottawa-1	5		5		5	
Ottawa-2		4		4		4
Mississauga-1		4		4		4
Mississauga-2	2		2		2	
Brampton-1		1		1		1
Brampton-2			4			4
Hamilton-1			3			3
Hamilton-2	3			3		
London-1		3			3	
London-2	13					
Markham-1		11				
Markham-2			10			
Vaughan-1	5					
Vaughan-2					2	
Windsor-1						2
Windsor-2						1
Total monthly workload	**28**	**43**	**24**	**32**	**12**	**39**

Table 5: Current schedule of maintenance services.

Question

Can your team help the distributor find a new 6-month schedule that minimizes the highest total monthly workload? Note that since a 6-month schedule will roll over for subsequent 6-month periods, the services with frequency 6, 3, and 2 should appear exactly 1, 2, and 3 times, respectively, in your schedule.

Answer

Write your schedule in the table on the next page. The highest total monthly workload in your schedule is _____.

Service	**January**	**February**	**March**	**April**	**May**	**June**
Toronto-1						
Toronto-2						
Ottawa-1						
Ottawa-2						
Mississauga-1						
Mississauga-2						
Brampton-1						
Brampton-2						
Hamilton-1						
Hamilton-2						
London-1						
London-2						
Markham-1						
Markham-2						
Vaughan-1						
Vaughan-2						
Windsor-1						
Windsor-2						
Total monthly workload						

3 Collectively Paying the Bill [100 marks]

Ten friends are at a restaurant enjoying dinner after a long day of volunteering for TORCH. It is time to pay the bill, which is, with tip and HST included: $175.00. The restaurant's debit and credit card machines are broken. Therefore, they all must pay by cash. Each person has a certain number of denominations as listed in Table 6 below.

Seat	**Name**	**Denominations**					
		$20.00	$10.00	$5.00	$2.00	$1.00	$0.25
1	Tony	2	0	0	0	0	1
2	Owain	1	2	1	0	0	0
3	Rachel	0	4	0	0	0	2
4	Chris	0	0	2	2	8	8
5	Houra	2	1	0	0	2	0
6	Iman	1	1	1	0	0	5
7	Shefali	5	3	1	0	3	0
8	Grace	1	6	1	2	3	0
9	Ryan	0	2	2	2	1	0
10	Taewoo	1	5	1	0	6	7

Table 6: Each person's number of denominations.

Assumptions:

- The group sits in a circle, counterclockwise, in the order given in the table above, so that Tony sits to Taewoo's right.
- The bill tray starts at seat 1. It is passed from one person to another counterclockwise. When the bill tray arrives at a person, he or she may put or take as much money as necessary.
- No one can exchange cash with another person except through the bill tray (i.e., if Chris puts $5.00 in the tray, Ryan may exchange that for two $2.00 coins and one $1.00 coin when he receives the tray.)
- Every time the bill tray arrives at a person, the maximum number of denominations he or she can put down is 5. For example, putting down two $20.00 bills and three $10.00 bills is allowed, but three $2.00 coins and three $1.00 coins is not.

Question

Is there a solution where everyone eventually pays equally ($17.50)? If so, minimize the number of times the bill tray must rotate around the circle.

Answer

Fill in the actions of each person during each rotation of the bill tray. For example, if in one rotation, Tony puts two $10.00 and takes one $5.00, write +2 in $10.00 row and -1 in $5.00 row. Add more tables if necessary.

Rotation 1	**Tony**	**Owain**	**Rachel**	**Chris**	**Houra**	**Iman**	**Shefali**	**Grace**	**Ryan**	**Taewoo**
$20.00										
$10.00										
$5.00										
$2.00										
$1.00										
$0.25										

Rotation 2	**Tony**	**Owain**	**Rachel**	**Chris**	**Houra**	**Iman**	**Shefali**	**Grace**	**Ryan**	**Taewoo**
$20.00										
$10.00										
$5.00										
$2.00										
$1.00										
$0.25										

Rotation 3	**Tony**	**Owain**	**Rachel**	**Chris**	**Houra**	**Iman**	**Shefali**	**Grace**	**Ryan**	**Taewoo**
$20.00										
$10.00										
$5.00										
$2.00										
$1.00										
$0.25										

Rotation 4	**Tony**	**Owain**	**Rachel**	**Chris**	**Houra**	**Iman**	**Shefali**	**Grace**	**Ryan**	**Taewoo**
$20.00										
$10.00										
$5.00										
$2.00										
$1.00										
$0.25										

Rotation 5	**Tony**	**Owain**	**Rachel**	**Chris**	**Houra**	**Iman**	**Shefali**	**Grace**	**Ryan**	**Taewoo**
$20.00										
$10.00										
$5.00										
$2.00										
$1.00										
$0.25										

Rotation 6	**Tony**	**Owain**	**Rachel**	**Chris**	**Houra**	**Iman**	**Shefali**	**Grace**	**Ryan**	**Taewoo**
$20.00										
$10.00										
$5.00										
$2.00										
$1.00										
$0.25										

Rotation 7	**Tony**	**Owain**	**Rachel**	**Chris**	**Houra**	**Iman**	**Shefali**	**Grace**	**Ryan**	**Taewoo**
$20.00										
$10.00										
$5.00										
$2.00										
$1.00										
$0.25										

Rotation 8	**Tony**	**Owain**	**Rachel**	**Chris**	**Houra**	**Iman**	**Shefali**	**Grace**	**Ryan**	**Taewoo**
$20.00										
$10.00										
$5.00										
$2.00										
$1.00										
$0.25										

4 Fleet of Xandor [150 marks]

Suppose you are the admirals of the powerful galactic navy of Xandor. Your galaxy has four planets A, B, C, and D and three star neighbors X, Y, and Z. The map of Xandor's planets and its star neighbors is shown in Figure 2.

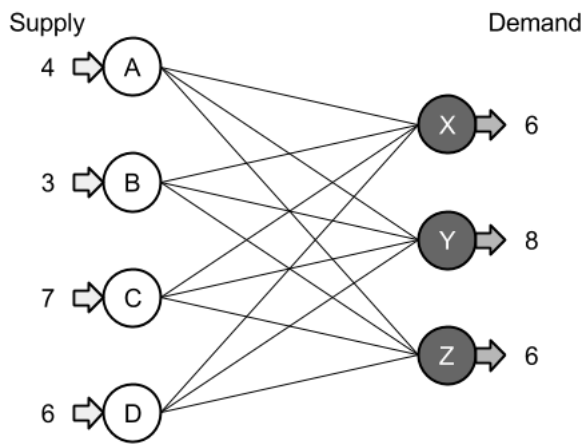


Figure 2: The map of Xandor's planets and its star neighbors.

Enemy battleships in stars X, Y, and Z threaten to invade Xandor. To defend against the enemy, you must send your ships from your planets to enemy stars. The minimum number of ships required to defeat a threat in a star is denoted as star demand in Figure 2. For example, you must bring at least 6 ships from your planets to defeat the star X threat. The number of ships that you have in planets A, B, C, and D are 4, 3, 7, and 6, respectively, denoted as planet supply in Figure 2. The fuel costs ($) to send one ship from the planets to the stars are given in Table 7.

From \ To	X	Y	Z
A	5	7	8
B	5	6	6
C	8	10	8
D	6	7	3

Table 7: The fuel costs ($) to send one ship from the planets to the stars.

Let f_{AX} denote the number of ships that you send from supply planet A to demand star X. The cost of sending f_{AX} ships equals $\$5 \times f_{AX}$. Define $f_{AY}, f_{AZ}, \ldots, f_{DY}, f_{DZ}$ similarly. Note that you cannot send fractional number of ships from planets to stars.

Question 1

You would like to overwhelm the enemy forces with all of your ships at the minimum possible fuel cost. Decide on the number of ships that you will send from each planet to each star such that no ships are left on the supply planets, all enemy threats are defeated, and the total fuel cost is minimized.

Answer 1

Write your answer below.

f_{AX} =_____

f_{AY} =_____

f_{AZ} =_____

f_{BX} =_____

f_{BY} =_____

f_{BZ} =_____

f_{CX} =_____

f_{CY} =_____

f_{CZ} =_____

f_{DX} =_____

f_{DY} =_____

f_{DZ} =_____

Total fuel cost = $_____

Nothing Lost, Nothing Gained

Experienced admirals know that overwhelming the enemy to guarantee victory is not always possible. Furthermore, battles are chaotic and the outcome of a battle is not known in advance. However, experienced tacticians are able to follow the ebb and flow of their previous decisions and make good educated guesses on battle outcomes.

As admirals of Xandor, you have a team of expert tacticians who can provide you with information on the chance of winning a battle. You know that your chance of winning increases by sending more ships to stars. However, it is impossible to win all fights. Therefore, you should decide which systems to fight for and which systems to yield to the enemy.

Since losing battles may be a necessary sacrifice to ensure winning the war, you might decide to send fewer ships than required to a star system. Recall that the demand of each star in Figure 2 represents the number of ships required to be sent to the star. In the first part of the question, a battle is won if the required number of ships is sent to a star system. However, in this part, it is not possible to guarantee victory if the required number of ships is sent to a star.

Let F_X, F_Y, and F_Z be the total number of ships that you send to stars X, Y, and Z, respectively. That is, $F_X = f_{AX} + f_{BX} + f_{CX} + f_{DX}$. Table 8 shows the chance of winning the battles in each enemy star based on the total number of ships sent, and the value ($) you gain if your chance of winning is 1 (value of winning). The value that you obtain from fighting battles in each star then equals the multiplication of the chance of winning and the value of winning. For example, if you send 5 ships to star X, you obtain $0.25 \times 200 = \$50$.

Star	**Chance of Winning**	**Value of Winning ($)**
X	$\frac{F_X}{20}$	200
Y	$\frac{F_Y^3}{9000}$	250
Z	$1 - \left[\frac{F_Z - 20}{20}\right]^2$	200

Table 8: Chance of winning and value of winning the fights in different stars.

Question 2

Your goal is to win the overall war and conserve fuel resources, leaving your fleet in a good state. Decide on the number of ships that you will send from each planet to each star. Your final score equals the total value obtained minus the total fuel cost. The total value obtained equals the sum of the values you obtain from fighting battles in the three stars.

Answer 2

Write your answer below.

f_{AX} =_____

f_{BX} =_____

f_{CX} =_____

f_{DX} =_____

F_X =_____

f_{AY} =_____

f_{BY} =_____

f_{CY} =_____

f_{DY} =_____

F_Y =_____

f_{AZ} =_____

f_{BZ} =_____

f_{CZ} =_____

f_{DZ} =_____

F_Z =_____

Total value obtained = $_____

Total fuel cost = $_____

Final score =_____

Instructions

- This booklet will be collected at 3:45PM.
- Please first write your team number on each page of this booklet. Note that failure to do so may result in your answers getting lost and your team receiving no marks for the corresponding questions.
- For each question, there is space in this booklet for writing your answer. Please write your final solutions in the indicated places, in the format explained in the questions. Answers stated on any other papers will NOT be marked.

5 Soccer Penalty Kick [100 marks]

Your team is playing against **two other teams** in the TORCH 2015 Soccer tournament. In the middle of this session of questions, around 2:30PM, you will be asked to send one ambassador to meet with your opponents' ambassadors for at most 10 minutes. Your ambassador may choose to leave the meeting early, but cannot stay any later. **Make sure that your ambassador writes down your opponents' team numbers.**

The soccer game consists of a single round robin of penalty kicks (i.e., your team will have one penalty kick against each of your two opponent teams). The kicker has the choice of kicking either to the left or to the right. The goalie on the opposing team tries to block the soccer ball. A kick occurs very quickly; therefore, the goalie cannot react to the direction of the kick after the kick is made. He/she must predict the direction of the kick in advance and leap in that direction at the same time the kick is made. If the goalie's prediction is wrong, he/she will not block the ball. If the goalie's prediction is correct, he/she will block the ball.

The kicker's performance is not, however, as good as the goalie's. If the kicker kicks the ball to the left, the ball will always go in if the goalie on the opposing team leaped to the right. If the kicker kicks the ball to the right, there is a 70% chance that the ball will NOT go in even if the goalie on the opposing team leaped to the left.

Question

Choose a kick direction and a block direction against each of your opponents.

Answer

Write your answer below.

Opponent's Team #	**Kick Direction**	**Block Direction**

6 Sworn Ark Offline Character Building [150 marks]

In the Sworn Ark Offline game, you create a powerful character that will fight a series of bosses. Each boss behaves very differently. Therefore, a powerful character should be flexible enough to handle as many bosses as possible.

Character Building

Figure 1 is a graph detailing how a character can be constructed for the battle.

To create your character, you start from the black circle S and choose at most 13 other circles on the graph. Each circle may only be chosen **once** and a circle can be chosen if it is connected (by a line) to at least one of the circles that have already been selected. For example, to include circle 13 in the character, circle 12 must have been chosen beforehand. You can construct your character only once to fight all of the bosses. In other words, you **cannot** change your character by adding or removing circles between bosses.

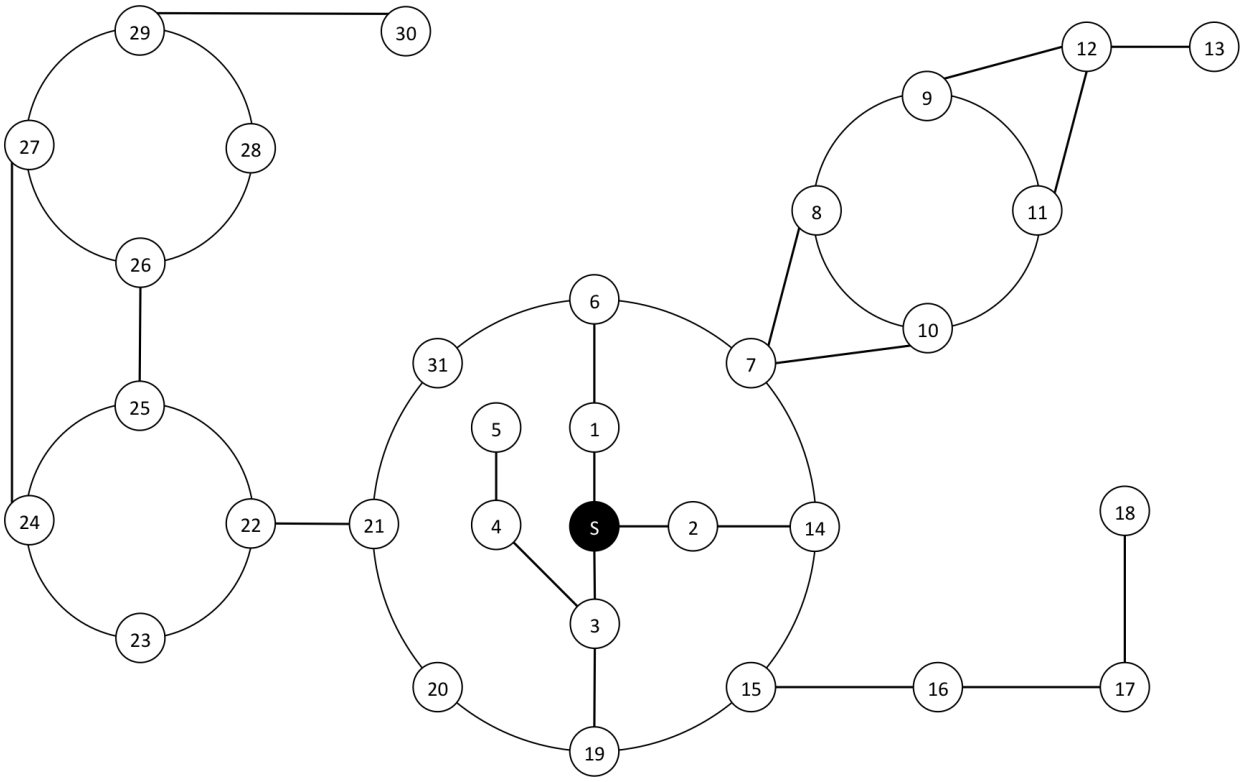


Figure 1: Character building graph. Start at circle S, choose at most 13 other circles to create your character.

The physical, mental, and magical prowess of your character in a fight are defined by Attack Power (A), Battle Experience (B), and Casting Power (C) features, respectively. Your character at the starting circle S has 0 points in each of these three features. Each circle can improve your character's skills in a specific way. The contributions of the circles to the character's features are defined in Table 1 below.

#	**Increase**	#	**Increase**	#	**Increase**
1	+6A	12	+4B and +20%B	23	+5A
2	+6B	13	+20% of total score for 1 fight of choice	24	+6A
3	+6C	14	+4B	25	+3C
4	+2 to all features	15	+2B	26	+3C and +10%C
5	+10% to all features	16	+2B	27	+5A
6	+4A	17	+2B	28	+7C and +10%C
7	+3B	18	Automatically win 1 fight of choice	29	+20%A and +20%B
8	+4B	19	+4C	30	+30% to all features
9	+4B	20	+4C	31	+4A
10	+20%B	21	+3A and +3C		
11	+20%B	22	+3A and +3C		

Table 1: The contributions of the circles to the character's features.

To find your character's final number of points in features A, B, and C, you may use the following three-step process:

1. Add all the flat amounts (e.g., +6A) over the circles forming your character to find the total flat amount.
2. Add all the percentage (%) increases over the circles forming your character to find the total percentage increase.
3. Increase the total flat amount by the total percentage increase, rounding the resulting value **down** to the nearest whole number.

For example, assume circles 2, 7, 10, 11, 12, and 14 form your character. These circles correspond to (+6B), (+3B), (+20%B), (+20%B), (+4B and +20%B), and (+4B). For feature B, the total flat amount is 6 + 3 + 4 + 4 = 17 and the total percentage increase is 20 + 20 +20 = 60. The total flat amount of 17 then increases by 60%, resulting in 27.2, which is rounded down to 27. Therefore, your character has 0 points A, 27 points B, and 0 points C.

Fighting Bosses

Bosses must be fought one at a time and in a predetermined order. Your character is able to fight the next boss if the previous boss is defeated. Your goal is to beat as many bosses as possible.

Bosses behave differently. Some are easier to defeat by physical attacks and/or magical spells and some require a character with good battle experience. The score that your character has to fight a boss with is a function of the character's three feature points, calculated using a specific scoring function. To defeat a boss, your character score should be greater than or equal to the boss score threshold. Table 2 below presents the bosses, the scoring functions, and the score thresholds.

Boss	**Scoring Function**	**Score Threshold**
1	$2A^3 + 4B + C$	13,500
2	$B^2 + C^2$	1,512
3	$\frac{B+C}{A}$	1
4	$(\frac{B}{A})^3 + C$	36
5	$\min(A, B, C)$	15
6	$A + B + C$	82
7	$20A + B^3$	16,259
8	$\frac{A}{2} + 3B + C$	100

Table 2: Scoring functions and score thresholds of eight different bosses.

The order in which you will fight the bosses is the same as the order the bosses appear in Table 2. For example, your character with 0 points A, 27 points B, and 0 points C has a score of 108 to fight boss 1. Since its score is less than 13,500, it does not beat boss 1 and cannot proceed to the next boss. The total number of bosses that your character beats is therefore 0.

Question

Start from the black circle S in Figure 1 and choose at most 13 other circles to create your character in order to maximize the number of bosses defeated.

Answer

Fill in your answer below.

1	2	3	4	5	6	7	8	9	10	11	12	13

- If you choose circle 13, state here the boss where your character score will be increased: _____. (Leave this blank if you did not choose circle 13.)
- If you choose circle 18, state here the boss where your character automatically wins: _____. (Leave this blank if you did not choose circle 18.)

7 Good Deals for Meals [150 marks]

Suppose you are a university student on a budget, and you want to start cooking instead of going out for dinner all the time. You learn the recipes of six different meals you would like to eat (potentially multiple times!) on a weekly basis. The table below lists the amount of each ingredient (in hundreds of grams) required to prepare the meals. For example, you need 200 grams of rice, 600 grams of chicken, 25 grams of cabbage, 100 grams of broth, and 100 grams of sprouts to make Chicken Teriyaki.

Ingredients	**Meals**					
	A Chicken Teriyaki	**B** Beef Stew	**C** Beef Soup	**D** Chicken Soup	**E** Ground Meat	**F** Pasta
Rice	2	2	0	0	2	0
Noodles	0	0	3	3	0	4
Chicken	6	0	0	4	6	6
Beef	0	8	8	0	0	0
Cabbage	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1
Carrot	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	0	1
Potato	0	1	1	$\frac{1}{2}$	0	1
Broccoli	0	$\frac{1}{3}$	0	$\frac{1}{2}$	1	1
Broth	1	2	1	2	2	1
Sprouts	1	0	1	1	1	1

Table 3: Meals and their associated ingredients.

You check your local supermarket and note the weight (in hundreds of grams) and the cost of a package of each ingredient in the table below. For example, one package of rice costs \$10 and has 600 grams of rice.

	Rice	Noodles	Chicken	Beef	Cabbage	Carrot	Potato	Broccoli	Broth	Sprouts
Weight	6	6	12	16	1	1	1	1	8	5
Cost	\$10	\$6	\$18	\$12	\$4	\$2	\$2	\$2	\$6	\$4

Table 4: Weight (in hundreds of grams) and cost of one package of each ingredient.

Assumptions:

- You **cannot** buy fractions of the ingredient packages. For example, you cannot buy 1.5 packages of rice.

- You can buy ingredients at any time during the week.
- You would like to cook dinners only, i.e., 7 meals per week.
- You can buy and use the ingredients on the same day.

Questions – treat each question separately. The restrictions of the previous question(s) do not affect following question(s).

1. Consider a scenario where you have learned how to cook meals A, B, and C only; you have to prepare 7 meals; and you do not want to eat the same meals on consecutive days (the same meal two days in a row). Find a cooking schedule for the week which minimizes the total cost of ingredients purchased. As an example, ABCABCA is a possible cooking schedule, but it does not necessarily minimize the cost. You need to buy 2, 1, 2, 2, 3, 2, 4, 1, 2, and 1 packages of rice, noodles, chicken, beef, cabbage, carrot, potato, broccoli, broth, and sprouts, respectively, at the total cost of $128, for this schedule. Remember that it is impossible to buy fractions of ingredient packages.

2. Consider a scenario where you know how to cook all six meals; you have to prepare 7 meals; and you do not want to eat the same meals on consecutive days. Find a cooking schedule for the week which minimizes the cost of ingredients purchased.

3. You discover that carrot, potato, broccoli and sprouts rot (cannot be used for cooking) after two days. For example, if you buy them on Sunday, you cannot use them for cooking on Tuesday. Find a new cooking schedule for the week which minimizes the cost of ingredients purchased assuming that you know how to cook all six meals; you have to prepare 7 meals; and you do not want to eat the same meals on consecutive days.

Answers

1. Fill in the meals you plan to prepare for each day and the ingredients that must be purchased (e.g., if you bought one package of rice, fill in ×1 under the Rice column).

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday

Rice	Noodles	Chicken	Beef	Cabbage	Carrot	Potato	Broccoli	Broth	Sprouts
×	×	×	×	×	×	×	×	×	×

Total cost of ingredients purchased is $_____.

2. Fill in the meals you plan to prepare for each day and the ingredients that must be purchased. Total cost of ingredients purchased is $_____.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday

Rice	Noodles	Chicken	Beef	Cabbage	Carrot	Potato	Broccoli	Broth	Sprouts
×	×	×	×	×	×	×	×	×	×

3. Fill in the meals you plan to prepare for each day and the ingredients that must be purchased on each day. Total cost of ingredients purchased is $_____.

Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday

Ingredients / **Days**	Rice	Noodles	Chicken	Beef	Cabbage	Carrot	Potato	Broccoli	Broth	Sprouts
Saturday	×	×	×	×	×	×	×	×	×	×
Sunday	×	×	×	×	×	×	×	×	×	×
Monday	×	×	×	×	×	×	×	×	×	×
Tuesday	×	×	×	×	×	×	×	×	×	×
Wednesday	×	×	×	×	×	×	×	×	×	×
Thursday	×	×	×	×	×	×	×	×	×	×
Friday	×	×	×	×	×	×	×	×	×	×

8 Wind Farm [100 marks]

To increase future power supply, the Optimal Power Generation (OPG) company has initiated a wind farm project near Toronto. A wind farm is a group of wind turbines that convert the kinetic energy from the wind into electrical power.

After the initial inspection, OPG has identified 25 potential locations for wind turbines. Each potential location is represented by a 1×1 square cell in Figure 2. The (x, y) values in each cell show the cost ($K) of building a turbine in the cell and the amount of power generated (MW), respectively. For example, it costs OPG $3K to build a turbine in the top-left cell, and 8MW power is generated as a result.

(3,8)	(3,7)	(2,4)	(11,21)	(6,13)
(14,31)	(13,27)	(20,44)	(12,25)	(7,16)
(4,8)	(15,31)	(13,25)	(12,24)	(2,5)
(7,15)	(11,22)	(15,36)	(12,22)	(4,9)
(10,22)	(7,16)	(8,18)	(12,23)	(2,4)

Figure 2: Potential locations of wind turbines.

OPG has a total budget of $50K for the wind farm project and can build at most one turbine in each cell.

Questions

1. OPG would like to know where to place the turbines to maximize the total amount of power generated, while meeting the budget constraint. Which cells does your team recommend that OPG select for building the turbines? The total amount of power generated is the sum of the amount of power generated in the cells selected.

2. Let's assume that, **at the same cost**, OPG can build a large turbine in a 2×2 square block instead of building four regular turbines in four 1×1 square cells. The large turbine generates 10MW more power than the the total amount of power generated by the corresponding four regular turbines. For example, it costs OPG $3 + 3 + 14 + 13 = \$33$K to build a large turbine in the top-left 2×2 square block, generating $(8 + 7 + 31 + 27) + 10 = 83$MW power. With the possibility of building large turbines, which blocks and cells does your team recommend that OPG select for building the large and the regular turbines in order to maximize the total

amount of power generated? Note that the total available budget is still $50K and each cell can be used for building at most one turbine, either large or regular.

Answers

1. Mark with × the cells in Figure 3 that your team recommends for building the turbines. The total amount of power generated is _____ MW.

(3,8)	(3,7)	(2,4)	(11,21)	(6,13)
(14,31)	(13,27)	(20,44)	(12,25)	(7,16)
(4,8)	(15,31)	(13,25)	(12,24)	(2,5)
(7,15)	(11,22)	(15,36)	(12,22)	(4,9)
(10,22)	(7,16)	(8,18)	(12,23)	(2,4)

Figure 3: Recommended locations for wind turbines.

2. Mark with * and ×, respectively, the blocks and the cells in Figure 4 that your team recommends for building the large and the regular turbines. The total amount of power generated is _____MW.

(3,8)	(3,7)	(2,4)	(11,21)	(6,13)
(14,31)	(13,27)	(20,44)	(12,25)	(7,16)
(4,8)	(15,31)	(13,25)	(12,24)	(2,5)
(7,15)	(11,22)	(15,36)	(12,22)	(4,9)
(10,22)	(7,16)	(8,18)	(12,23)	(2,4)

Figure 4: Recommended locations for large and regular wind turbines.

9 Prime and Unique

Select a prime number between 2 and 40 (inclusive). The selected prime number will be the number of marks awarded to your group if no other group selects the same prime number. If two or more groups select the same prime number, all of these groups will receive no marks. What will your prime and unique choice be?

Answer

Your prime and unique number is _____.