The Operations Research Challenge 2014

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Instructions

- You have 2 hours and 30 minutes to write the first part of the competition. This booklet will be collected at 12:45 pm.
- Please first write your group number on each page of this booklet. Note that failure to do so may result in your answers getting lost and your group receiving no marks for the corresponding questions.
- For each question, there is space in this booklet for writing your answer. Please write your final solutions in the indicated places, in the format explained in the questions. Answers stated on any other papers will NOT be marked.

1 Automated External Defibrillator (AED) Placement [200 marks]

Cardiac arrest usually occurs suddenly and unexpectedly and is fatal to the victim if help does not arrive within a few minutes. A life-saving device called an automated external defibrillator (AED) is available in many public places to help cardiac arrest victims. An AED is a portable electronic device that automatically diagnoses cardiac rhythms and delivers a shock to correct abnormal activity in the heart. AEDs are installed in public places such as airports, shopping malls, schools and subway stations and can be used by bystanders in case of a cardiac arrest emergency. A defibrillatory shock is most effective when delivered within three minutes. Considering the set-up time of the device, this means that an AED is useful for cardiac arrests happening at most 100 meters away from the AED.

Figure 1 is a map of part of the Moss Park neighborhood in Toronto. This area historically has the highest number of cardiac arrests. The City of Toronto would like to put an AED in the area to prevent deaths from cardiac arrest. The main question is: where should the AED be placed? To answer this question, the University of Toronto Operations Research team determined the potential locations of 58 cardiac arrests in the Moss Park area (red dots on the map) using statistical models based on historical data. They then divided the area into equal squares, each measuring 25 meters by 25 meters. Now they need your help!

Question 1: Your first mission is to find the best location for an AED to save as many cardiac arrest patients as possible. You can save a patient only if you have an AED within 100 meters. You may locate an AED **on any corner (intersection) point of the grid** shown on the map (only corner point locations are allowed). Distances between corner points should be calculated using direct paths (i.e., Euclidean distance). For example, a direct path between location (23, P) and location (26, R) has length $\sqrt{(2 \times 25)^2 + (3 \times 25)^2} = 90.14$ meters. You need to state both the location of the AED and the corresponding number of lives it can save.

Answer:

- Coordinates of the best location: (,   ). Please also indicate the location you pick on the map, using a **star** symbol.
- Number of lives saved:

Question 2: The city council gave another thought to the problem and decided to fund two AEDs instead of one! What are the best locations for these two AEDs? (Both locations can be different from the location you picked in question 1.) How many lives will be saved in total from the two locations?

Answer:

- Coordinates of the two best locations: (,) and (,). Please also indicate the locations you pick on the map, using two **filled circles**.
- Number of lives saved:

Question 3: What is the minimum number of AEDs needed to save all 58 cardiac arrest patients?

Answer:

Question 4: In reality, the survival chance of a cardiac arrest victim decreases rapidly for every minute of delay in defibrillation. Therefore, the closer the AED is to the patient, the higher the survival probability. Assume that the probability of saving a patient is 1 if the distance between an AED and the patient is less than or equal to 50 meters, and 0.5 if the distance is greater than 50 meters but less than or equal to 100 meters. If only one AED is available, where should it be placed to maximize the expected number of lives saved? Expected number of lives saved is defined as $1\times$(number of patients within 50 meters from the AED) + $0.5\times$(number of patients at distance greater than 50 meters but less than or equal to 100 meters from the AED). How many lives can be saved with probability 1, and how many can be saved with probability 0.5?

Answer:

- Coordinates of the best location: (,). Please also indicate the location you pick on the map, using a **triangle** symbol.
- Number of lives saved with probability 1:
- Number of lives saved with probability 0.5:

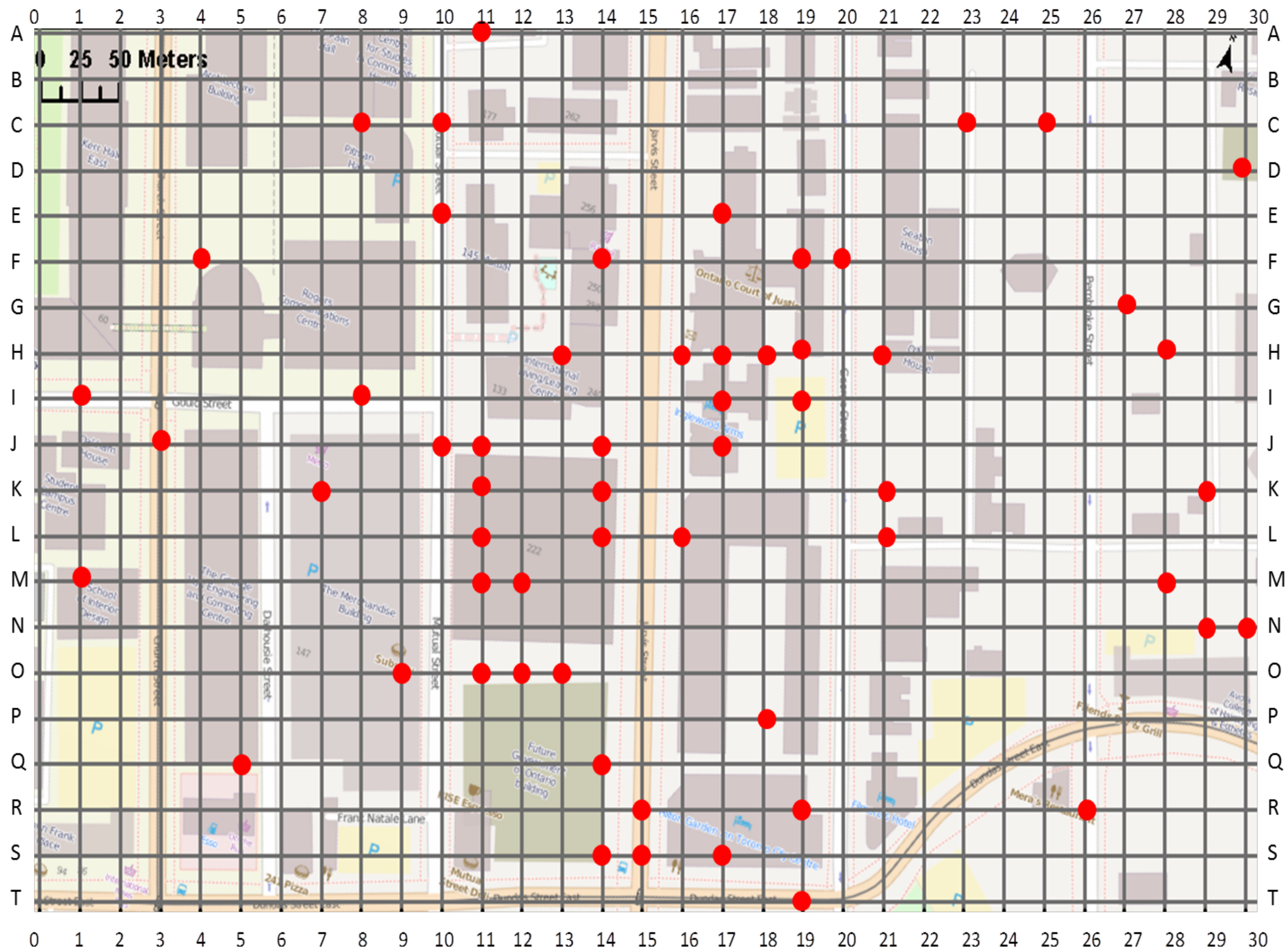

Figure 1: Part of the Moss Park neighborhood in Toronto.

2 Party! [100 marks]

Olivier would like to celebrate his graduation by inviting 6 of his close friends over for dinner. However, his dinner table can fit at most 5 people including himself. He decides that he will have two parties, and thus he needs a way to divide his friends into two groups. He also would like to avoid awkward silences, so he decides to group friends based on the TV series that they watch (so that there will be lots to talk about!). Help Olivier group his friends so that the number of shared TV show interests is maximized! (Note that Olivier's own TV preferences are not considered.)

Olivier can invite a friend to more than one dinner. However, if two people already had dinner together, they will not talk to each other about their favourite TV shows after the first time. His friends and their favourite TV shows are listed below (a 1 denotes the fact that the person watches the corresponding show).

Friend/TV Show*	MF	SP	H	HIMYM	TWD	TBBT	BB
Peter			1	1	1		1
Kimia	1	1		1		1	1
Velibor		1		1	1	1	
Auyon			1		1	1	1
Curtiss		1	1		1		1
Derya	1			1	1	1	

Table 1: TV show preferences of Olivier's friends. *MF: Modern Family, SP: South Park, H: Homeland, HIMYM: How I Met Your Mother, TWD: The Walking Dead, TBBT: The Big Bang Theory, BB: Breaking Bad.

You can evaluate the quality of a group by finding the number of common TV shows for every pair of friends who haven't yet had dinner together, and then summing these values over all pairs. Note that every pair contributes points to the dinner score at most once! Help Olivier choose two groups to maximize the overall score.

Example: One option for Olivier is to have the following two parties. Note that not all of his friends are invited in this example!

<u>Party #1: Peter, Kimia, and Velibor</u>

Peter & Velibor: share HIMYM and TWD -> 2 points
Peter & Kimia: share HIMYM and BB -> 2 points
Kimia & Velibor: share SP, HIMYM, and TBBT -> 3 points

Therefore, dinner party 1 has a total score of 7.

Party #2: Kimia, Velibor, and Derya

Kimia & Velibor: already spent dinner party 1 talking to each other -> 0 points
Kimia & Derya: share MF, HIMYM, and TBBT -> 3 points.
Velibor & Derya: share HIMYM, TWD and TBBT -> 3 points

Therefore, dinner party 2 has a total score of 6.

The total score for these two dinner parties is 13.

Answer:

Party #1:

Party #2:

Total score:

3 Scheduling Events at the Winter Olympics [200 marks]

Imagine that you go back in time and are hired by the organizing committee of the 2014 Winter Olympics in Sochi, Russia. Your main responsibility is the scheduling of events at three Olympic venues: the Iceberg Skating Palace, the Laura Biathlon and Ski Complex, and the Sanki Sliding Center.[1] Tables 2, 3 and 4 provide lists of sporting disciplines, together with their duration (in hours, stated in square brackets) for each of the three venues.

Question 1: Your first goal is to build a schedule that minimizes the number of days needed to complete all of the given events.

Discipline	**Event 1**	**Event 2**	**Event 3 (Medal Event)**
Pairs Figure Skating		Short Program [2]	Free Program [4]
Ice Dance Figure Skating	Original Dance [2]	Short Dance [2]	Free Dance [2]
Women's Short Track Speed Skating	Qualifying [2]	Semi-finals [2]	Finals [2]
Men's Short Track Speed Skating	Qualifying [2]	Semi-finals [2]	Finals [2]
Relays	–	–	All Rounds [6]

Table 2: Events at the Iceberg Skating Palace.

Discipline	**Event 1**	**Event 2**	**Event 3 (Medal Event)**
Sprint Free Style	–	–	Race [6]
Men's Mass Start Biathlon	–	–	Race [2]
Women's Mass Start Biathlon	–	–	Race [1]

Table 3: Events at the Laura Biathlon and Ski Complex.

Discipline	**Event 1**	**Event 2**	**Event 3 (Medal Event)**
Women's Bobsleigh	–	Rounds 1 & 2 [2]	Rounds 3 & 4 [3]
Men's Bobsleigh	–	Rounds 1 & 2 [2]	Rounds 3 & 4 [2]
Men's Luge	–	–	All Rounds [5]
Women's Luge	–	–	All Rounds [4]
Team Relay Luge	–	–	All Rounds [3]

Table 4: Events at the Sanki Sliding Center.

[1] http://en.wikipedia.org/wiki/2014_Winter_Olympics#Venues

Here are some additional restrictions set out by the organizing committee that you need to take into account when building the schedule:

1. The total number of hours per day per venue that is available for scheduling (14) may not be exceeded.
2. Events may start only on the hour (i.e., an event may be scheduled to start at 1:00, but not 1:15, 1:30, etc.).
3. Events scheduled at the same venue may not overlap in time (in other words, at most one event may be occurring at a given venue at any point in time).
4. Events of the same discipline cannot be held on the same day. For example, you are not allowed to schedule the pairs figure skating free program on the same day as the pairs figure skating short program.
5. For a given discipline, event 1 (if any) has to be held before event 2, and event 2 (if any) has to be held before event 3. For example, the pairs figure skating short program has to be held before the pairs free program; men's bobsleigh rounds 1 & 2 have to be held before men's bobsleigh rounds 3 & 4, etc.
6. Medals are awarded after event 3 of each discipline: hence, these are called medal events. Medal events in two or more disciplines may not overlap in time (even if they occur at different venues!). For example, the pairs figure skating free program may not overlap in time with the sprint free style race.
7. Women's and men's mass start biathlon cannot be held on the same day.
8. Men's and women's luge have to be completed before team relay luge. Moreover, team relay luge may not be scheduled on the same day as men's luge, and team relay luge may not be scheduled on the same day as women's luge, since the relay competition includes both men and women participants.

<u>Please state your schedule in the table provided on the next page.</u>

Day	Day 1			Day 2			Day 3		
Time/Venue	Skating Palace	Biathlon/Ski Complex	Sliding Center	Skating Palace	Biathlon/Ski Complex	Sliding Center	Skating Palace	Biathlon/Ski Complex	Sliding Center
8 – 9									
9 – 10									
10 – 11									
11 – 12									
12 – 13									
13 – 14									
14 – 15									
15 – 16									
16 – 17									
17 – 18									
18 – 19									
19 – 20									
20 – 21									
21 – 22									

Day	Day 4			Day 5			Day 6		
Time/Venue	Skating Palace	Biathlon/Ski Complex	Sliding Center	Skating Palace	Biathlon/Ski Complex	Sliding Center	Skating Palace	Biathlon/Ski Complex	Sliding Center
8 – 9									
9 – 10									
10 – 11									
11 – 12									
12 – 13									
13 – 14									
14 – 15									
15 – 16									
16 – 17									
17 – 18									
18 – 19									
19 – 20									
20 – 21									
21 – 22									

Question 2: After you create the first schedule, the organizing committee realizes that it did not tell you the full story. In particular, the durations of events given in Tables 2, 3 and 4 are only rough estimates. The actual durations of the events are **unknown**: they may depend on the weather, damage of the tracks/trails during the event and the efficiency of the maintenance crew; they also include any immediate setup before the event and clean-up after the event. For example, if there is not enough real snow prior to a race at the Laura Biathlon and Ski Complex, the maintenance crew will need to make new snow, delaying the actual start of the competition; at the Sanki Sliding Center, if a bobsleigh makes a significant scratch on the surface of the bobsleigh track during competition, repair will have to be performed, delaying the continuation of the competition.

New data for the durations of events The organizing committee therefore gives you more data: for each of the events in Tables 5 and 6, a **set of potential durations** is given. For example, the sprint free style race may take 6 hours or 7 hours; the men's mass start biathlon may take 2 hours or 5 hours. Note that you are not given any information about how likely these duration scenarios are.

Discipline	**Event 1**	**Event 2**	**Event 3 (Medal Event)**
Sprint Free Style	–	–	Race $\{6, 7\}$
Men's Mass Start Biathlon	–	–	Race $\{2, 5\}$
Women's Mass Start Biathlon	–	–	Race $\{1, 4\}$

Table 5: Events at the Laura Biathlon and Ski Complex; for each event, a set of potential durations is given.

Discipline	**Event 1**	**Event 2**	**Event 3 (Medal Event)**
Women's Bobsleigh	–	Rounds 1 & 2 $\{2, 7\}$	Rounds 3 & 4 $\{3, 6\}$
Men's Bobsleigh	–	Rounds 1 & 2 $\{2, 7\}$	Rounds 3 & 4 $\{2, 3\}$
Men's Luge	–	–	All Rounds $\{5, 7\}$
Women's Luge	–	–	All Rounds $\{4, 6\}$
Team Relay Luge	–	–	All Rounds $\{3, 8\}$

Table 6: Events at the Sanki Sliding Center; for each event, a set of potential durations is given.

The duration of the original dance program happening at the Iceberg Skating Palace is $\{2, 3\}$. The durations of all other events at the Iceberg Skating Palace, given in Table 2, are all known with certainty, i.e., they are assumed to always have the duration given in Table 2.

Modified restrictions Rules 2 – 8 from part (a) still apply. However, rule 1 is changed to the following: the total number of hours per day per venue may be exceeded, until time 8:00 the next day (the start of the next Olympic day). The latest time an event *may be scheduled to start* is 21:00. A schedule in which all events scheduled on the same day are not finished by 8 am of the next Olympic day is *not allowed* (you will receive 0 marks for such a schedule).

Explanation of Delays The true start times and finish times of events will be known only when your schedule is implemented (in real time). Using your schedule *with the true durations* may result in delays of some of the events (see examples below). Also, due to these delays, in reality events may start later than 21:00 (even if they are *scheduled* to start at or earlier than 21:00).

Cost Calculation Your task is to construct a schedule with the new data in order to minimize the total cost while ensuring that all events get completed. The total cost of the schedule consists of two parts, a fixed cost based on the total number of days in your schedule, and an "uncertainty" cost:

- Fixed cost = number of days in your schedule × 100.
- The true uncertainty cost can be known only when your schedule is implemented and the events are completed. In particular, using your schedule *with the true durations* may result in delays of some of the events (see examples below). For every hour of such delay, your schedule incurs a cost of 10. Furthermore, for every hour of going past 22:00, a penalty cost of 30 is incurred. Thus, uncertainty cost = (number of hours of delay) × 10 + (number of hours going past 22:00) × 30.

Example:

Optimistic Schedule: You are optimistic that men's and women's bobsleigh rounds 3 & 4 will last 2 and 3 hours, respectively, and so you construct a schedule in which men's bobsleigh is scheduled from 8 to 10 on day 2, and women's bobsleigh is scheduled from 10 to 13 on the same day. Women's mass start biathlon is scheduled to start at 13. The corresponding schedule is shown in Table 7.

Day	**Day 2**		
Time/Venue	**Skating Palace**	**Biathlon/Ski Complex**	**Sliding Center**
8 − 9			Men's Boblsleigh
9 − 10			Men's Boblsleigh
10 − 11			*Women's Bobsleigh*
11 − 12			*Women's Bobsleigh*
12 − 13			*Women's Bobsleigh*
13 − 14		Women's Mass Start Biathlon	

Table 7: An optimistic schedule.

Suppose that in reality, when the schedule is implemented, men's bobsleigh takes 3 hours and women's bobsleigh takes 3 hours. Thus, when your schedule is executed, both women's bobsleigh and women's biathlon are delayed by 1 hour (compared to your schedule), as shown in Table 8. We refer to this schedule as a *realized* schedule, or a *schedule realization*. Since women's bobsleigh is delayed by 1 hour and women's biathlon is delayed by 1 hour, and assuming there is nothing scheduled after women's biathlon, your schedule incurs a penalty cost of 20.

Day	**Day 2**		
Time/Venue	**Skating Palace**	**Biathlon/Ski Complex**	**Sliding Center**
8 − 9			Men's Boblsleigh
9 − 10			Men's Boblsleigh
10 − 11			Men's Boblsleigh
11 − 12			*Women's Bobsleigh*
12 − 13			*Women's Bobsleigh*
13 − 14			*Women's Bobsleigh*
14 − 15		Women's Mass Start Biathlon	

Table 8: Realization of the optimistic schedule.

Conservative Schedule: Suppose instead that you are conservative and assume that men's bobsleigh will last 3 hours and women's bobsleigh will last 6 hours. You therefore construct a schedule where men's bobsleigh is scheduled from 8 to 11 on day 2, women's bobsleigh is scheduled from 11 to 17 on the same day, and that women's biathlon is scheduled to start at 17. The corresponding schedule is shown in Table 9. Suppose again that in reality, men's bobsleigh takes 3 hours and women's bobsleigh takes 3 hours. The realization of the conservative schedule is shown in Table 10.

Day	**Day 2**		
Time/Venue	**Skating Palace**	**Biathlon/Ski Complex**	**Sliding Center**
8 − 9			Men's Boblsleigh
9 − 10			Men's Boblsleigh
10 − 11			Men's Boblsleigh
11 − 12			*Women's Bobsleigh*
12 − 13			*Women's Bobsleigh*
13 − 14			*Women's Bobsleigh*
14 − 15			*Women's Bobsleigh*
15 − 16			*Women's Bobsleigh*
16 − 17			*Women's Bobsleigh*
17 − 18		Women's Mass Start Biathlon	

Table 9: A conservative schedule.

In this case, your schedule does not incur any penalties, since women's bobsleigh and women's biathlon will start at exactly the time when they were scheduled. However, since women's bobsleigh takes 3 hours, there is now some idle time in the schedule before women's biathlon starts (3 hours). That is, being conservative implies a potential increase in the total number of days necessary to complete all events.

Day	**Day 2**		
Time/Venue	**Skating Palace**	**Biathlon/Ski Complex**	**Sliding Center**
8 − 9			Men's Boblsleigh
9 − 10			Men's Boblsleigh
10 − 11			Men's Boblsleigh
11 − 12			*Women's Bobsleigh*
12 − 13			*Women's Bobsleigh*
13 − 14			*Women's Bobsleigh*
14 − 15			
15 − 16			
16 − 17			
17 − 18		Women's Mass Start Biathlon	

Table 10: Realization of the conservative schedule.

Your task is to construct a schedule that will minimize the total cost, which is the sum of the fixed cost and the uncertainty cost... without knowing the actual durations!

Wait, one more thing: you can ask the organizing committee the true durations of up to 3 events. However, each time you ask, you get a penalty of 12 added to the total cost of your schedule.

Please state your schedule in the tables provided on the next two pages.

Day	Day 1			Day 2			Day 3		
Time/Venue	Skating Palace	Biathlon/Ski Complex	Sliding Center	Skating Palace	Biathlon/Ski Complex	Sliding Center	Skating Palace	Biathlon/Ski Complex	Sliding Center
8 – 9									
9 – 10									
10 – 11									
11 – 12									
12 – 13									
13 – 14									
14 – 15									
15 – 16									
16 – 17									
17 – 18									
18 – 19									
19 – 20									
20 – 21									
21 – 22									
22 – 23									
23 – 24									
24 – 1									
1 – 2									
2 – 3									
3 – 4									
4 – 5									
5 – 6									
6 – 7									
7 – 8									

Day	Day 4			Day 5			Day 6		
Time/Venue	Skating Palace	Biathlon/Ski Complex	Sliding Center	Skating Palace	Biathlon/Ski Complex	Sliding Center	Skating Palace	Biathlon/Ski Complex	Sliding Center
8 – 9									
9 – 10									
10 – 11									
11 – 12									
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1 – 2									
2 – 3									
3 – 4									
4 – 5									
5 – 6									
6 – 7									
7 – 8									

4 Electricity Transmission [150 marks]

On December 22nd, 300,000 Toronto residents lost power as a severe ice storm struck the central and eastern portions of Canada. Many residents were left without power in the following days. Due to limited resources, Toronto city workers could not bring power back to all residents in a timely manner. Although it is not possible to restore power everywhere immediately, it is important to give power to as many people as possible, as early as possible. In Figure 2, a portion of Toronto's electrical grid is shown. The squares represent power stations and the circles are neighbourhoods consisting of many homes. The population of a neighbourhood is given by the number in the circle. The lines in the figure are electrical transmission lines. In order for a neighbourhood to be powered, it must be connected to at least one power station. A connection means that a path can be made between the neighbourhood (circle) and a power station (square) by travelling along the transmission lines. During the storm, falling branches have broken many transmission lines. In Figure 2, a red line represents a broken electrical transmission line. Therefore, any path that uses a red line is not a proper electrical connection for a neighbourhood.

You are given two teams of electricians that will travel around Toronto and repair the broken transmission lines. Each team will work a 12-hour shift. Assume that it takes exactly 2 hours for one team to repair one line. Hence, the shift is divided into 6 time periods. For each time period, you must decide which two transmission lines should be repaired in order to restore power to as many people as possible, as quickly as possible. You can indicate which lines are repaired and the order of these repairs by writing a number on the line you wish to repair. The number 1 beside a line will indicate that the line is repaired during time period 1. Note that there should be exactly two 1's since you have two teams of electricians. The number 2 means that the line is repaired in time period 2, etc.

To determine the quality of your schedule, you need to calculate the total population that is connected to a power station at the end of each two-hour time period and then sum these values over all time periods. Please indicate your solution on Figure 3.

Example: Figure 2 shows an example solution for the first two time periods.

- Time period 1: electricity is restored for a population of $3+3+5 = 11$. The total population that now has power is 11.
- Time period 2: electricity is restored for a population of $3 + 9 = 12$. The total population that now has power is $11 + 12 = 23$.
- The total score after two periods is $11 + 23 = 34$.

Answer:

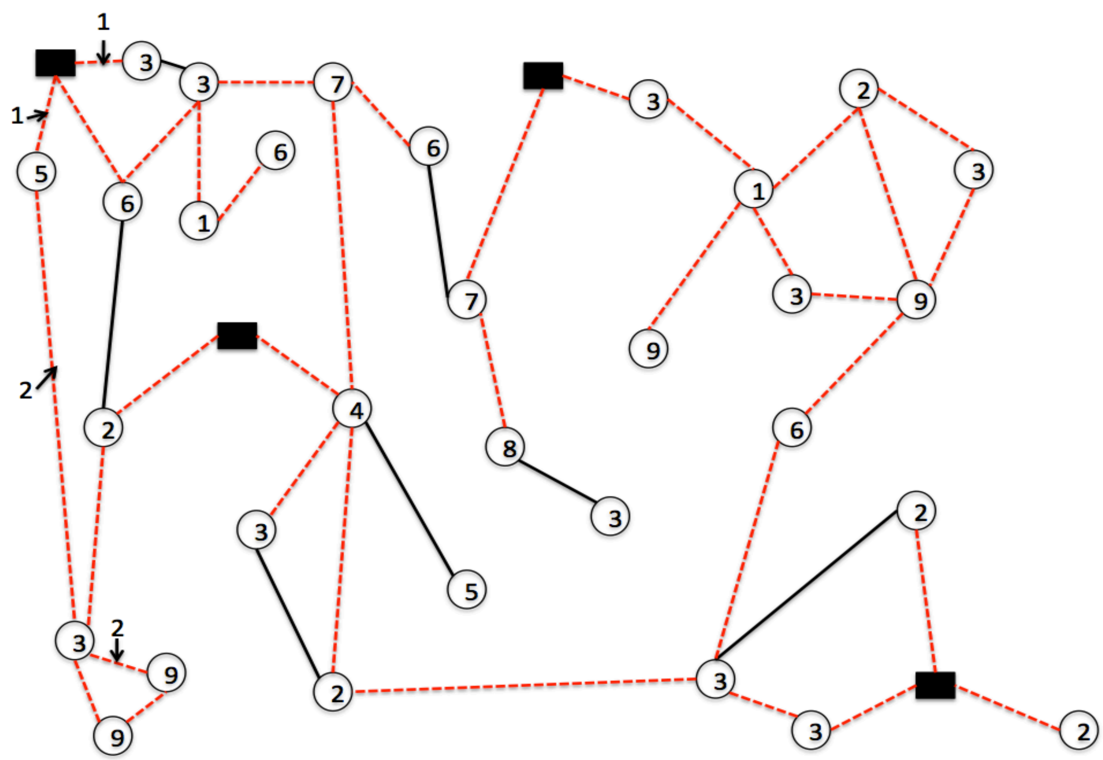

Figure 2: Part of the Toronto electricity grid and an example solution.

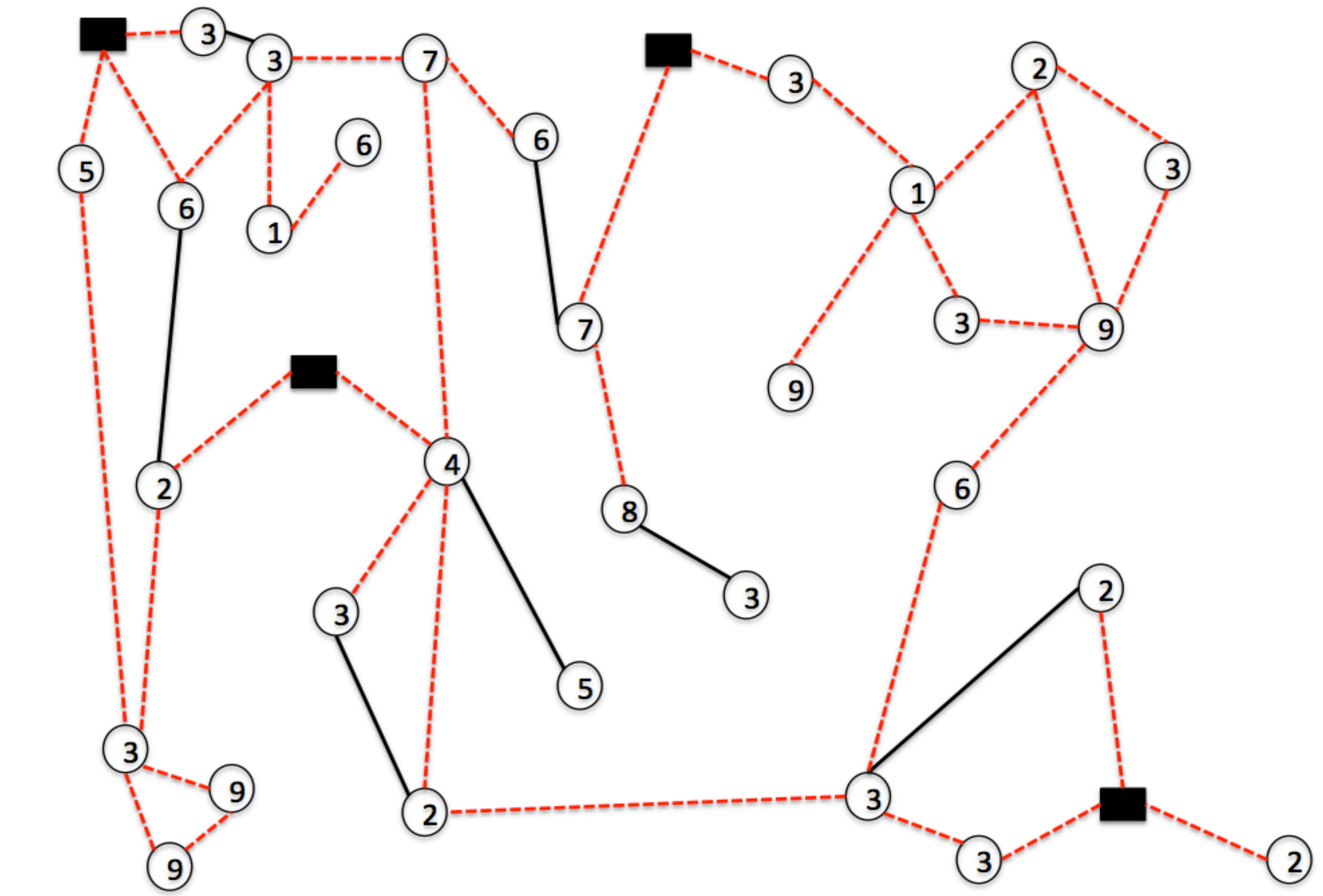

Figure 3: Please state your solution here.

Instructions

- This booklet will be collected at 3:45 pm.
- Please first write your group number on each page of this booklet. Note that failure to do so may result in your answers getting lost and your group receiving no marks for the corresponding questions.
- For each question, there is space in this booklet for writing your answer. Please write your final solutions in the indicated places, in the format explained in the questions. Answers stated on any other papers will NOT be marked.

5 Microchip Technology for Cancer Diagnosis [200 marks]

It has been discovered that cancer spreads by circulating tumor cells (CTCs) from one organ to other, non-adjacent, organs. CTCs are therefore the key to tracking cancer progression.[1]

To monitor the cancer status of a patient, CTCs should be isolated from the patient blood for further examinations. Since the number of CTCs is very low in the blood cells, the main challenge to CTC examination is associated with the development of technologies that can efficiently isolate CTCs from the blood.

A recently-developed technology in CTC examination uses microchips to capture CTCs from the blood. The isolated tumor cells are then cultured in controlled conditions for further experimentation. The Ontario Institute for Cancer Research (OICR) has a project on the characterization of a new drug for cancer treatment and would like to use the new CTC examination technology. OICR has hired your team to help them find the optimal plan for buying the new technology.

OICR has provided your team with the following information:

- Five different types of microchips are available in the market with various prices, speeds, and efficiencies, as shown in Table 1. The efficiency and the speed of a microchip represent the percentage of CTCs captured and the time that it takes to finish the capturing process, respectively. For example, if the total available number of CTCs in a sample of blood is 500, microchip A captures 75 ($500\times0.15=75$) tumor cells in 5 minutes.
- OICR works on blood samples with 500 CTCs and needs to capture at least 350 of the tumor cells within 120 minutes for any further measurements.
- Microchips are used one at a time. In the other words, microchips cannot be used in parallel. For example, assume that there are two microchips of type A. If the first one is used at time 0, then 75 ($500\times0.15=75$) CTCs are captured. The second one may then be used at time 5, capturing 63 ($[500-75]\times0.15=63.75$) additional CTCs.
- Once a chip is used, it should be cleaned before it can be used again. The cleaning process takes 60 minutes.

OICR would like your team to determine how many microchips of each type should be bought and how they should be sequenced so that at least 350 CTCs are captured within 120 minutes and the total cost is minimized. An example solution is given below. Please state your solution on the following page, in the same format as the example solution.

[1] This question is motivated by the paper of Sheng et al. (2014). Complete reference for the paper: W. Sheng, O. O. Ogunwobi, T. Chen, J. Zhang, T. J. George, C. Liu and Z. H. Fan. Capture, release and culture of circulating tumor cells from pancreatic cancer patients using an enhanced mixing chip, *Lab on a Chip*, 2014 (14), 89–98.

Microchip Type	Price	Speed (min)	Efficiency (%)
A	$4,000	5	15
B	$2,600	7	13
C	$2,200	8	10
D	$1,500	10	7
E	$800	15	4

Table 1: The price, speed and efficiency of different microchip types.

Example Solution:

The number of chips to buy is stated in the following table:

Microchip Type	Number to buy
A	4
B	0
C	0
D	0
E	0

Total cost: 4 × $4000 = $16000

Let's call the chips A1, A2, A3, A4. The order in which OICR should use these chips is A1-A2-A3-A4-A1-A2-A3-A4, and the corresponding schedule is:

Time		Chip used	Number of CTCs captured
From	To		
0	5	A1	75
5	10	A2	63
10	15	A3	54
15	20	A4	46
65	70	A1	39
70	75	A2	33
75	80	A3	28
80	85	A4	24

Total CTCs captured: 362

Total time: 85

Answer:

Microchip Type	**Number to buy**
A	
B	
C	
D	
E	

Total cost: __________

Time		**Chip used**	**Number of CTCs captured**
From	**To**		

Total CTCs captured: __________

Total time: __________

6 Movie Theatre Seating Arrangement [150 marks]

The TORCH movie theatre has recently opened and its business is going well. However, today the booking system is not running properly and the seating information of some customers is lost. Therefore, customers who originally booked tickets as a group might not be able to sit together due to limited seat availability. The manager has decided on the following rules and discounts:

1. A group of 7 people may be split into a group of 4 people and a group of 3 people: each person in the group of 4 receives a 20% discount, and each person in the group of 3 receives a 40% discount.
2. A group of 5 people may be split into a group of 3 people and a group of 2 people: each person in the group of 3 receives a 40% discount, and each person in the group of 2 receives a 70% discount.
3. A group of 4 people may be split into 2 groups of 2 people: each person in the original group receives a 70% discount.
4. Rules 1 to 3 state the only allowed ways of separating a group.
5. If a group gets seats without being separated, all members of the group pay the full ticket price.
6. If a group cannot get seats without being separated, and also cannot get seats by being separated according to rules 1 to 3, then all people in the group receive a 100% refund.

The price of a ticket for each person is $10. For example, if you separate a group of 7 people, you will earn $4\times10\times0.8 + 3\times10\times0.6 = 50$ dollars from this group.

Figure 1 shows the seating chart of the movie theatre. The grey boxes indicate the seats that are known to be currently occupied.

Question: There are 5 groups of 7 people, 7 groups of 5 people and 8 groups of 4 people that need to find seats. The manager asks for your help to make the profit as high as possible by optimizing the assignment of seats to the groups. Please state the seats assigned to the groups in Tables 3–5. Please also state the total profit.

Example for filling out the tables:

Group of 4	Split?	Seats
Group 1	yes	(A5,A6), (B11,B12)

Table 2: Example for filling out the tables.

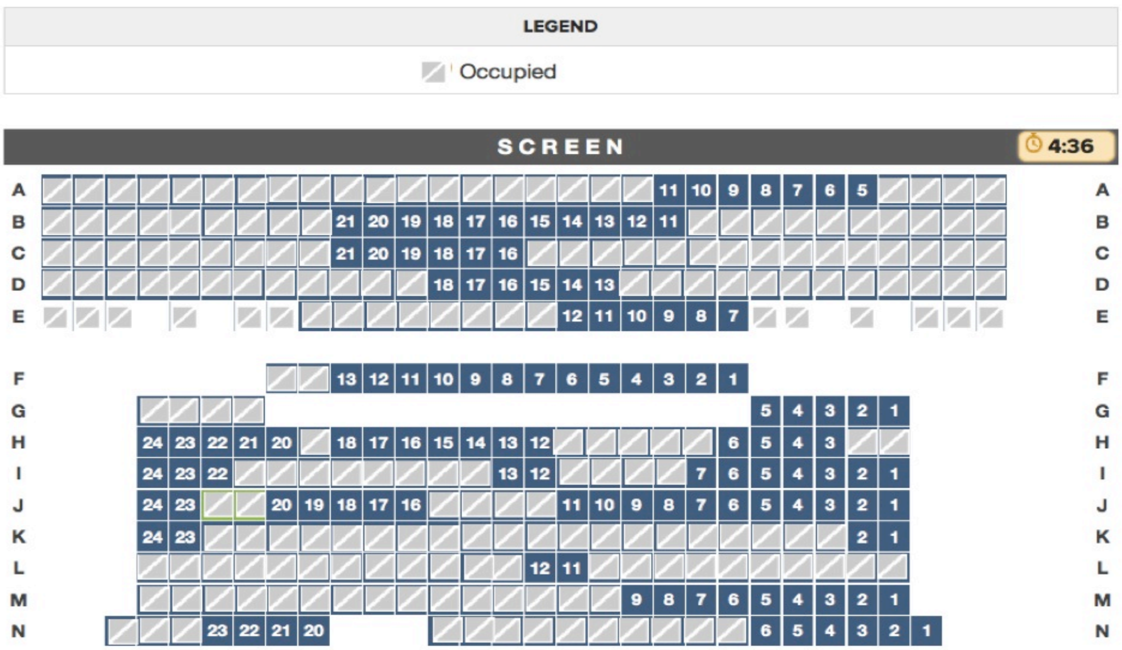

Figure 1: Seating chart of the movie theatre with some of the seats already occupied.

Answer:

Total Profit: ---------

Groups of 4	**Split?**	**Seats**
Group 1		
Group 2		
Group 3		
Group 4		
Group 5		
Group 6		
Group 7		
Group 8		

Table 3: Assignment of seats to groups of 4.

Groups of 7	**Split?**	**Seats**
Group 1		
Group 2		
Group 3		
Group 4		
Group 5		

Table 4: Assignment of seats to groups of 7.

Groups of 5	**Split?**	**Seats**
Group 1		
Group 2		
Group 3		
Group 4		
Group 5		
Group 6		
Group 7		

Table 5: Assignment of seats to groups of 5.

7 Money Multiplier [150 marks]

Suppose that your uncle and your aunt have successful careers in operations research and would like to encourage you to take the same career path. With that in mind, they have designed a money game to spark your interest in operations research.

Every day, your uncle will give you a \$1 baseline for a total duration of 7 days, from day 1 to day 7. Starting on day 2, your aunt will offer you a "multiplier" based on the day's number. So, if it is day 2, you can get a times-2-money-multiplier and thus get \$2; if it is day 3, you can get a times-3-money-multiplier and thus get \$3, etc. Once you accept a multiplier, you have to keep it until you accept another multiplier (if any). Every day during which you are in possession of a multiplier, the amount of money you get is multiplied by the value of the multiplier. When you accept a new multiplier, the previous one is replaced by the new one entirely (i.e., the multipliers *do not* add up).

Example 1: If you could receive a money-multiplier each day, how much money would you make at the end of 7 days?

- Answer: 1+2+3+4+5+6+7 = 28

Example 2: Suppose you are only allowed to receive the times-3-money-multiplier (on day 3). How much money would you make at the end of 7 days?

- Answer: 1+1+3+3+3+3+3 =17

Question 1: If you could receive exactly one money-multiplier, which one would you take?

Answer:

- Money-multiplier:
- Total amount of money at the end of 7 days:

Question 2: If you could receive exactly two money-multipliers, which ones would you take?

Answer:

- Money-multipliers:
- Total amount of money at the end of 7 days:

Question 3: The next time you play the game, your uncle extends it to 21 days. If you could choose exactly two money-multipliers, which ones would you choose?

Answer:

- Money-multipliers:
- Total amount of money at the end of 21 days:

Question 4: To make the problem more difficult, your uncle next decides that you can get square-multipliers. A square-multiplier on day 2 will give you 2^2 dollars, a square-multiplier on day 3 will give you 3^2 dollars, etc. Assume the game lasts 21 days. As before, when you accept a new square-multiplier, the previous one is replaced by the new one entirely (i.e., the multipliers *do not* add up). If you could receive exactly one square-multiplier, which one would you take?

Answer:

- Square-money-multiplier:
- Total amount of money at the end of 21 days:

Question 5: Assume again (as in the previous question) that you have 21 days and get square-multipliers. If you could choose exactly two square-multipliers, which ones would you choose?

Answer:

- Two square-money-multipliers:
- Total amount of money at the end of 21 days:

8 An Optimal Figure Skating Program [200 marks]

Your task is to design an optimal short program for a figure skater, assuming that the short program has 8 required elements: 3 jumps, 3 spins and 2 step sequences.[2] Table 6 below specifies the jump, spin and step sequence types that are available. Each jump, spin and step sequence type may be performed at most once.

The difficulty of an element (move) is measured by its base grade of execution (GOE) value (which is then used for judging the program). The base GOE values are provided in Table 6.[3] However, the more difficult an element is, the greater the probability that the skater will not execute it perfectly. Therefore, Table 6 also states the **expected** GOE scores for the given skater. That is, the expected GOE score takes into account the probability that the given skater may not perform the element perfectly. The quality of the program you design is measured in terms of the **expected** GOE values.

Jump Name	**GOE (B, E)**	**Spin Name**	**GOE (B, E)**	**Sequence Name**	**GOE (B, E)**
Triple Toeloop (3T)	(4.10, 2.87)	Upright (USp3)	(1.90, 1.71)	Step Level 1 (StSq1)	(1.80, 1.80)
Triple Salchow (3S)	(4.20, 2.94)	Sit (SSp3)	(2.10, 1.89)	Choreographic (ChSq)	(2.00, 1.80)
Triple Loop (3Lo)	(5.10, 2.55)	Camel (CsP3)	(2.30, 1.61)	Step Level 2 (StSq2)	(2.60, 2.34)
Triple Flip (3F)	(5.30, 2.12)	Layback (LSp3)	(2.40, 1.20)	Step Level 3 (StSq3)	(3.30, 2.64)
Triple Lutz (3Lz)	(6.00, 1.80)			Step Level 4 (StSq4)	(3.90, 1.95)
Triple Axel (3A)	(8.50, 1.70)				

Table 6: Element Names and their Base (B) and Expected (E) Grade of Execution (GOE) Scores.

First vs. second half of the program The expected GOE score of an element is influenced by the position of the element in the program. Positions 1 to 4 correspond to the first half of the program, and positions 5 to 8 correspond to the second half of the program. Performance of difficult moves in the second half of the program earns bonus points from the judges, but decreases the probability of success of those moves due to the skater's fatigue after the half-way point of the program. Hence, the expected GOE of an element may increase or decrease from the first to the second half of the program. Both subsequent tables have two values for each move, one corresponding to the case when the element is scheduled in the first half of the program, and the other corresponding to the case when the element is scheduled in the second half of the program.

Easier element before a harder element Performing an easier move before a harder one increases the probability of success of the second (harder) move (due to the increase in the skater's

[2] General figure skating info taken from: "Short Program (figure skating)", Wikipedia, http://en.wikipedia.org/wiki/Short_program_(figure_skating)

[3] "Scale of Values, Levels of Difficulty and Guidelines for marking Grade of Execution", International Skating Union, Communication No. 1724, http://www.usfsa.org/content/ISU%201724%20S&P%20SOV_GOE_Levels%20of%20difficulties%202012-2013.pdf

confidence). Table 7 shows the expected GOE values for each element, assuming that an easier move is performed directly before it. Note that "easier" is evaluated in terms of the **base GOE** given in Table 6.

Example 1: we schedule 3T as the first element of the program and 3S as the second

- since 3T is the first scheduled element of the program, we obtain an expected GOE of 2.87 (Table 6);
- according to the base GOE values in Table 6, 3T is easier than 3S, and hence we obtain 2.98 for 3S from Table 7;
- the total expected value for the two moves is 2.87 + 2.98 = 5.85.

Harder element before an easier element On the contrary, performing a harder move directly before an easier one leads to earning bonus points from the judges. Table 8 shows the expected GOE values for each element, assuming that a harder move is performed directly before it. Note that "harder" is evaluated in terms of the base GOE given in Table 6.

Example 2: we schedule 3S as the first element of the program and 3T as the second

- since 3S is the first scheduled element, we obtain an expected GOE of 2.94 (Table 6);
- according to the base GOE values in Table 6, 3T is easier than 3S, and hence we get 2.88 for 3T from Table 8;
- the total expected value for the two moves is 2.94 + 2.88 = 5.82.

Question: Your task is to design a program (a sequence of 8 moves) in order to maximize the expected quality of the program. The expected quality of a program is simply the sum of the expected GOE values (which can be found in Tables 6, 7 and 8). Please state the sequence, the expected GOE values corresponding to each element and the total expected GOE value in the spaces provided below.

Answer:

Sequence: ______ ______ ______ ______ ______ ______ ______ ______

GOE Expected Values: ______ ______ ______ ______ ______ ______ ______ ______

Total GOE Expected Value: ____________

Jump	GOE (1st, 2nd)	Spin	GOE (1st, 2nd)	Sequence	GOE (1st, 2nd)
3T	(3.00, 3.01)	USp3	(1.72, 1.91)	StSq1	(1.89, 1.92)
3S	(2.98, 3.09)	SSp3	(1.90, 1.89)	ChSq	(1.89, 1.79)
3Lo	(2.57, 2.53)	CsP3	(1.64, 1.66)	StSq2	(2.36, 2.37)
3F	(2.14, 2.15)	LSp3	(1.27, 1.25)	StSq3	(2.67, 2.89)
3Lz	(1.90, 1.87)			StSq4	(1.97, 1.92)
3A	(1.80, 1.56)				

Table 7: Expected GOE values for each element, assuming that this element is directly preceded by an easier element. “1st” corresponds to the value that would be achieved in the first half of the program, and “2nd” to the value that would be achieved in the second half of the program.

Jump	GOE (1st, 2nd)	Spin	GOE (1st, 2nd)	Sequence	GOE (1st, 2nd)
3T	(2.88, 2.98)	USp3	(1.72, 1.91)	StSq1	(1.87, 1.90)
3S	(2.96, 3.07)	SSp3	(1.90, 1.89)	ChSq	(1.88, 1.78)
3Lo	(2.57, 2.53)	CsP3	(1.63, 1.65)	StSq2	(2.35, 2.36)
3F	(2.14, 2.15)	LSp3	(1.23, 1.21)	StSq3	(2.66, 2.88)
3Lz	(1.87, 1.84)			StSq4	(1.96, 1.91)
3A	(1.75, 1.51)				

Table 8: Expected GOE values for each element, assuming that this element is directly preceded by a harder element. “1st” corresponds to the value that would be achieved in the first half of the program, and “2nd” to the value that would be achieved in the second half of the program.

9 Bonus Question

You have the chance to increase your mark for a question that you think you didn't do well on during the TORCH competition. You may choose any of the questions from this year's competition and have your mark for that question be set to

$$\max\{\text{your original mark, average mark}\},$$

where *average mark* is the average mark on this question, computed over all groups participating this year. For example, if you get 50 marks for the Money Multiplier question and the average mark (over all the groups, including yours) is 80, then your final mark for this question is adjusted to 80.

Your decision: Below please circle the number of the question for which you want to use the above formula for computing its mark.

Question	Name	Total Marks
1	Automated External Debrillator (AED) Placement	200
2	Party	100
3	Scheduling Events at the Winter Olympics	200
4	Electricity Transmission	150
5	Microchip Technology for Cancer Diagnosis	200
6	Movie Theatre Seating Arrangement	150
7	Money Multiplier	150
8	An Optimal Figure Skating Program	200