Instructions

- You have 2 hours and 15 minutes to write the first part of the competition.
- Please first write your group number on each page of both the question and the answer sheets. Note that failure to do so may result in your answers getting lost and your group receiving no marks for the corresponding questions.
- For each question, there is an answer sheet. Please write your final solutions on the answer sheets in the format explained in the questions. Answers written on any other papers will NOT be marked.
- You have the choice of submitting the answer to Question 3 (Castle Location) at the end of the second part of the competition.

The	Operations	Research	Challenge	2013:	Part I
1110	Operations	1 COD COI CII	CHAHOHE	-010.	1 010 1

Group: _____

1 Hunger Games [130 marks]

In the world created by Suzanne Collins, the evil rulers of the Panem nation force its twelve districts to send a teenage boy or girl (as a tribute) for the annual Hunger Games. The tributes must fight to the death in an arena, with a sole survivor remaining. This year's arena is a vast wilderness area.¹

Katniss Everdeen is taken from District 12 to compete in this year's Hunger Games. She has survived the game so far, but her survival backpack is now empty. On the third day of the game, 30 minutes before sunset, she fortunately finds a map of her surrounding area (Figure 1) marked with the locations of 20 different survival items. Since it is too dangerous to travel around in the dark, Katniss has 30 minutes to fill her backpack. However, she has two problems:

- 1) She can carry at most 30 pounds.
- 2) She knows that some items are more important to her survival than others.

Therefore, she must choose her items wisely. Each item is given a survival value, and she wants to maximize the total survival value. The weights and the survival value of the items are given in Table 1.

The current location of Katniss is marked with "S" on the map. Katniss can move directly one square up/down/left/right and each move takes exactly 1 minute; she cannot move diagonally. If there is an item in a square, she has the choice of picking up the item or of leaving it. Picking up the item does not take any time.

Help Katniss choose the items and the path she will take to gather these items to maximize the total survival value of the items she collects. After exactly 30 minutes, she will immediately stop and hide, without obtaining the remaining items. Draw the path that Katniss takes on the map (given in the answer sheet) and circle each item she picks up along her path.

Item	Weight	Survival value	Item	Weight	Survival value
	(lbs)			(lbs)	
1. Sleeping bag	9	15	11. Arrows	5	10
2. Dried beef strips	5	14	12. Flint	6	9
3. Nuts	3	10	13. Towel	1	2
4. Bow	10	25	14. Tent	25	50
5. Night vision glasses	10	15	15. Coiled wire	7	5
6. Slingshot	10	3	16. Bottle of iodine	5	12
7. Peeta's medicine	15	30	17. Extra pair of socks	4	10
8. Flashlight	3	5	18. Matches	1	2
9. Knife	3	7	19. Mockingjay Pin	2	2
10. Water	5	15	20. Rain jacket	12	25

Table 1: List of survival items, their weights, and their survival value

¹Collins, S. (2008). The Hunger Games. New York: Scholastic Press.

							Τ		T				1	
S														
14		11		3							20			
							9							
													6	
			10						7					
					16			18				5		
	12													
				13			1							
										8				
		2				19								4
														17
										15				

Figure 1: The map of the surrounding area

2 Super Snowstorm in Canada [200 marks]

Canada is hit by a super snowstorm and the official reports have declared it a major disaster. To investigate the situation across the country, the prime minister has decided to send four of the best explorers from Ottawa to the capitals of the 10 provinces and the 3 territories.



Figure 2: The capital cities of the provinces and the territories

a) [100 marks] Each explorer is responsible for gathering the information on the impact of storm in different provinces and territories so that the government can better manage resources for helping the storm victims. The government needs this information as soon as possible. The prime minister asks you to lead this mission, and your job is to assign thirteen capital cities to four explorers such that each city is visited by only one explorer, and to schedule a route for each explorer that starts from Ottawa, visits each assigned city once, and then returns to Ottawa.

The storm has destroyed the major roads between cities. After some investigation on the new possible roads between cities, you find the travel times between Ottawa and the capital cities, and between cities, as given in Table 2 and Table 4, respectively. Note that the travel times between Ottawa and the capital cities are the same in both directions, but the travel times between any other two cities (Table 4) might be dependent on the direction.

Draw the route of each explorer on the map (given in the answer sheet) and compute the total traveling time for each explorer to minimize the longest time that an explorer takes to return to Ottawa. For example, consider two solutions, S_1 and S_2 , where the total travel times for four explorers are (20, 42, 31, 29) and (32, 40, 41, 34), respectively. The longest travelling time in S_1 is 42 for the second explorer, but it is 41 in S_2 for the third explorer. Given the goal of your mission, you prefer solution S_2 to solution S_1 .

	ON	$\overline{\mathbf{QC}}$	NS	NB	MB	BC	\mathbf{PE}	SK	AB	NL	YU	NT	NU
Ottawa	60	68	86	34	22	51	35	52	74	74	31	18	15

Table 2: Travel times between Ottawa and the capital cities of the 10 provinces and 3 territories

- b) [100 marks] Assuming that all explorers successfully return to Ottawa, the government then decides to send supplies to each capital. However, since most of the gasoline supplies are unreachable, there is only a total amount of 1000 liters of gasoline left in Ottawa. There are 4 types of trucks (A, B, C, D) available for delivery of supplies to cities, with the following characteristics:
 - 1) Each type of truck consumes 1 liter of gasoline per time unit. For example, any truck type needs 14 liters of gasoline to travel between ON and QC.
 - 2) Each truck type has a different fuel tank volume, as shown in Table 3. For example, a truck of type A can travel up to 100 time units because it has a 100-liter fuel tank.
 - 3) Some types of trucks can travel faster between certain cities. If the name of a truck appears in a square in Table 4, it can travel between the corresponding cities in half the time, but only in the specified direction. For example, the travel time from ON to NS is 17 for Truck B and 34 for all other trucks, but the travel time from NS to ON remains at 34 for all truck types (including type B). The travel times between Ottawa and the capital cities given in Table 2 remain the same for different types of trucks.

To minimize the longest time that any truck travels, your job is the following:

- Decide how many trucks of each type should be used for sending the supplies to all thirteen capital cities.
- Assign the thirteen cities to the trucks such that each city is visited by only one truck.
- Schedule a route for each truck that starts from Ottawa, visits each assigned city once, and returns to Ottawa.

In your solution, list the number of trucks of each type, specify the route that is traveled by each truck on the map (given in the answer sheet), and compute the total traveling time for each truck.

Truck type	A	В	С	D
Fuel tank volume	100	200	300	500

Table 3: Fuel tank volume of different types of trucks

To From	ON	\mathbf{QC}	NS	NB	MB	BC	PE	SK	AB	NL	YU	NT	NU
ON	-	14	$34_{ m B}$	$62_{\mathrm{B,C}}$	$76_{\mathrm{C,D}}$	$60_{ m D}$	64_{C}	$52_{ m D}$	$34_{ m D}$	30	$43_{ m D}$	31	50
\mathbf{QC}	14	-	$82_{ m D}$	$84_{ m D}$	$43_{ m C}$	33_{A}	$57_{\rm C}$	36_{D}	77_{C}	40	$15_{ m D}$	38	$43_{ m D}$
NS	34	$42_{ m D}$	-	25	$37_{\mathrm{B,D}}$	$55_{\rm C}$	$30_{\rm C}$	41	$31_{\scriptscriptstyle m D}$	$34_{ m D}$	$50_{\rm C}$	$17_{\scriptscriptstyle m D}$	9
NB	$62_{\rm C}$	$84_{\rm C}$	25	-	$61_{ m B,C}$	67	$70_{ m D}$	$62_{ m D}$	$52_{ m D}$	48	79	$70_{ m D}$	$47_{ m D}$
MB	76_{B}	$43_{ m C}$	37	$61_{\rm C}$	-	$83_{\rm C}$	$92_{ m A,D}$	55	88	55	89	55	46
вс	$60_{ m D}$	$33_{ m D}$	55	67_{C}	83_{B}	-	$35_{ m D}$	$43_{ m D}$	$54_{ m C}$	$94_{ m D}$	$67_{\rm C}$	38	59
PE	$64_{ m D}$	57	$80_{\rm C}$	$70_{ m A,C}$	$92_{\mathrm{C,D}}$	$35_{ m D}$	-	$23_{ m D}$	33	43	44_{B}	$54_{ m D}$	46
SK	52	36	$41_{\mathrm{C,D}}$	$62_{\mathrm{B,D}}$	$55_{\mathrm{C,D}}$	43	23	-	$35_{ m D}$	46	$55_{ m A,D}$	$47_{ m C}$	66
AB	$34_{ m D}$	$77_{\mathrm{C,D}}$	$31_{A,D}$	52	$28_{ m D}$	54_{B}	33	$35_{ m D}$	-	28_{C}	15	88	$61_{\rm C}$
NL	39	40	$44_{ m D}$	48	35	$44_{\rm C,D}$	$43_{ m D}$	$46_{ m D}$	18_{C}	-	36	$39_{\mathrm{B,D}}$	$56_{\rm C}$
YU	43_{D}	15	$50_{ m D}$	$79_{A,D}$	$29_{ m D}$	67	$44_{\rm C}$	55_{B}	$15_{\rm C}$	$36_{\rm C}$	-	79_{D}	47
NT	$61_{\scriptscriptstyle m D}$	88 _D	$77_{\scriptscriptstyle m D}$	$70_{\rm C}$	85	69	$91_{\rm C}$	$71_{\scriptscriptstyle m D}$	$80_{\rm C}$	19_{B}	$79_{\scriptscriptstyle m D}$	-	26
NU	$80_{ m D}$	$63_{ m D}$	$53_{\rm C}$	77	$66_{\rm C}$	$59_{ m D}$	$46_{\rm C}$	66_{B}	$61_{ m D}$	56_{A}	47	99_{D}	-

Table 4: Travel times between the capital cities of the corresponding provinces and territories

The	Operations	Research	Challenge	2013:	Part I
1110	Operations	1 COD COI CII	CHAHOHE	-010.	1 010 1

Group: _____

3 Castle Location [200 marks]

Your father, the king of Orland, has given you and three of your siblings a large piece of land to share. Different sections of this land are shown in Figure 3. Your father provides you with sufficient funds and manpower to build your own city and castle anywhere you like, all you need to do is tell him where and he will make it happen.

To ensure your city is prosperous, three useful resources are required. You wish to locate your city near a source of water (W), a forest (F) and a mine (M). These three resources are equally important and being too far from any of them will negatively affect your city. To make matters worse, your relationship with your siblings is quite temperamental. You all love each other, but your own prosperity is more important than the relationship you hold with them. You do not want to locate your city too close to the city of a sibling or else you will have to spend resources to hire spies and build defenses in case your sibling decides to attack you. You also do not want to be too far from your siblings, as this will make trading goods difficult.

Your task is to find a location for your city such that it is near enough to all natural resources and is in a good distance from your nearest sibling city. If you choose the same location as one of your siblings, your father will decide to build a city at that location and take it for himself, leaving you with nothing.

To define your distance to any resource or sibling city, you must count the number of squares you are away (only going up/down/left/right) from the closest square containing the resource/city in question. The size of a city is exactly one square. For example, if you decide to locate your city in square A-4, you will be 1 square away from the mine (up once), 5 squares away from water (down 5 times), and 13 squares away from the forest (right 9 times, down 4 times).

Using the following penalty formula, you can calculate how bad a location is:

Penalty =
$$d_W^2 + d_M^2 + d_F^2 + (16 - d_S)^2$$

where:

 d_W = distance to water d_M = distance to a mine d_F = distance to a forest d_S = distance to a sibling

Mark the location of your city on the map given in the answer sheet and clearly explain the reasons for your choice.

Attention:

- 1) We have assigned you siblings. You will receive the information on who your siblings are on a separate sheet. If you have not yet received it, please be patient we will bring it to you soon.
- 2) You have the choice of submitting your solution to this question at the end of the second part of the competition.

Marking: We will use the locations of the cities chosen by your three siblings to calculate the score of your city following the penalty formula. We will then sort all team scores in ascending order and will assign the corresponding marks between 0 and 200.

	A	В	C	D	\mathbf{E}	\mathbf{F}	G	Н	Ι	J	K	\mathbf{L}	M	N	О	P	Q	R	S	\mathbf{T}
1	М	M	М				W	W	W	W	W	W	W	W						W
2	М	M	М																	W
3	М	M																		W
4																				
5																				
6																				
7																				
8										F	F									М
9	W								F	F	F	F								М
10								F	F	F	F	F	F							М
11	М							F	F	F	F	F	F							
12	М								F	F	F	F								W
13	M									F	F									
14																				
15																				
16																				
17																				
18	W																		M	M
19	W																	M	M	M
20	W						W	W	W	W	W	W	W	W				М	M	М

Figure 3: Map of Orland

4 Lord of the Rings [150 marks]

You and your adversary have claims on 15 ancient rings. The rings will be placed on a 15-slot roulette wheel with one ring per slot. Your adversary will take the first turn and may pick any ring on the wheel. Each of you will then take turns to claim one ring at a time and can only claim rings that are adjacent to an empty slot. The value of each ring is listed in Table 5.

You and your adversary both want to maximize the total value of the claimed rings. You are assigned the duty of placing the rings on the roulette wheel before the first run such that your total collected value is maximized. Devise a placement of the rings on the roulette wheel given in the answer sheet and explain the reasons for your placement. As part of your answer, provide two scenarios that show the following:

- a) In the first scenario, your total collected value is maximized if your adversary claims the ring with the highest possible value in all her/his turns.
- b) In the second scenario, your total collected value is maximized if your adversary does NOT necessarily claim the ring with the highest possible value in all her/his turns.

For each of the scenario, clearly state your and your adversary's total collected value. Note that you cannot write the same scenario for both part a) and part b). The scenarios need to be different.

Marking: We will simulate your scenarios using the strategies given in part a) and part b) for your adversary. The score that you get for a scenario is the difference between your total collected value and your adversary's one. Your total score for this question is the sum of the scores for two scenarios. To get the final mark for this question, we will sort all team scores in decreasing order and will assign the corresponding mark between 0 and 150.

Attention: The optimal placement of the rings is the one which maximizes your total collected value regardless of which strategy your adversary is using. If your placement of the rings is the optimal one, you will be awarded **100 bonus marks**.

Name of the ring	Quantity	Value of the ring
Diamond (ring of status)	3	200
Moonstone (ring of intellect)	3	100
Iron (ring of poor engineers)	9	10
Total	15	990

Table 5: Values of different rings

Instructions

- You have 2 hours to write the second part of the competition.
- Please first write your group number on each page of both the question and the answer sheets. Note that failure to do so may result in your answers getting lost and your group receiving no marks for the corresponding questions.
- There are answer sheets for Questions 5, 6, and 7. Please write your final solutions on the answer sheets in the format explained in the questions. Answers stated on any other papers will NOT be marked.
- Please write the answer to Question 8, Prime and Unique, in the place assigned to it on the **question** sheet.
- If you have not submitted the answer to the Castle Location question, you should submit it at the end of this part of the competition.

The	Operations	Research	Challenge	2013:	Part II
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Group: _____

5 Robots -vs- Zombies! [270 marks]

The year? 2029. The location? U of T campus. The problem? Zombies!!

You are the lead engineer of a world-class robotics team at the university, and find yourself cornered, injured, and out of options. You *need* to get some help, and fast, but because of your injuries there's no chance you can navigate the zombie infested campus by yourself. Your only hope is to use your latest robot project, the Radioactive Unidirectional Myopic Bionic Android (RUMBA).

Your mission is to program RUMBA to navigate the terrain to get to a target location where the rest of your research team is hiding out. If successful, they will certainly come to your rescue (you do pay their salary after all!). The main problem with this plan is that you have very fuzzy intelligence on where the zombies actually are. Even worse, because of budget cutbacks, the sensors on RUMBA can only sense if a zombie is nearby, but not its exact location.

5.1 Example

The details on the rules for programming RUMBA are provided in the next two sections, but for now let's look at an example.

Assume that there are two possible scenarios for zombie locations (shown below), RUMBA will need to navigate a 6 by 6 grid, with a single obstacle (thick lines in the middle) and 4 zombies (\mathbf{Z}) that are scattered throughout. RUMBA will start its navigation from the square labeled with \mathbf{R} , indicating start, and your job is to program RUMBA such that it reaches the square labeled \mathbf{G} , indicating goal, regardless of which of the two potential scenarios is the real one.

Scenario 1:

R				
		Z	Z	
Z	Z			
				G

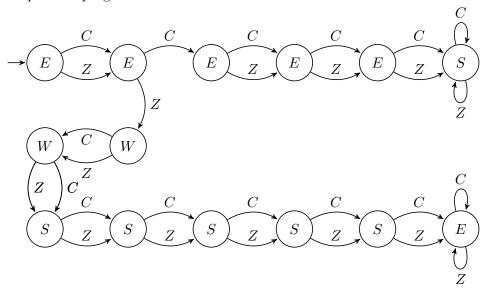
Scenario 2:

R		Z	
	_	Z	
	Z		
	Z		
			G

At each square, in the first step, RUMBA uses its sensors to identify zombies. It can sense a zombie if there is at least one directly beside the robot (not including diagonals), but cannot identify the zombie's exact location. The outcome of the first step can be either **Z**ombie or **C**lear. In the second step, after the outcome of sensing becomes clear, RUMBA needs to do a move to any one of **N**orth, **S**outh, **E**ast, or **W**est.

¹The government put more money into plant research in case of the zombie apocalypse – a huge mistake.

A possible program with which RUMBA reaches G is as follows:



RUMBA starts by going east twice (no matter what is sensed after the first move). If a zombie is detected, RUMBA will head back west to the start, and then go all the way south before heading all the way east again (this solves the second scenario). If RUMBA did not detect a zombie to begin with, it will continue all the way east before heading south (this solves the first scenario). Notice that the single program above solves both scenarios.

5.2 How to Program RUMBA

The way you program RUMBA is to construct *just one* Visual Plan Representation (VPR), an example of which is shown above. The VPR has two main components:

- 1) **Action nodes**: This tells RUMBA where to move, and it can be any one of **N**orth, **S**outh, **E**ast, or **W**est (see the circles in the example VPR above)
- 2) **Sensing links**: This is what RUMBA senses when looking for zombies, and it can be either **Z**ombie or Clear (see the edges in the example VPR above)

Remember, RUMBA will sense a zombie if there is at least one directly beside the robot (not including diagonals) and cannot tell in *which* direction the zombie is located if one is detected. There are further restrictions on the VPR that you *must* follow (otherwise RUMBA will blow up and you get 0 points):

- Every action node must have *exactly two* outgoing sensing links (one for \mathbf{Z} and one for \mathbf{C}). There can be any number of incoming links.
- You must designate a starting action node for RUMBA to begin moving (you will never start next to a zombie).

5.3 RUMBA Operation

The way RUMBA operates is by moving around the map based on the action nodes. In its starting location, RUMBA will move in the specified direction. It then senses if there is a zombie or not (\mathbf{Z} or \mathbf{C}), and the appropriate sensing link is followed to a new action node. These two steps repeat until either:

- 1. RUMBA reaches the goal location (winning!)
- 2. RUMBA runs into a zombie (zombies love to eat bionic robots)

There might be obstacles and walls in the environment (you will know where they are), but RUMBA is allowed to try and move into them (it just will not go anywhere). While it does not change the robot location, moving into a wall may change the action node currently used in the VPR. ²

5.4 Scenarios

According to information you have received over the campus-wide Zombie Emergency Broadcast (ZEB), there are 9 potential scenarios for the zombie locations. The scenarios can be found on the following page. Remember, you can use only *one* VPR for every scenario. Note that there are some scenarios that RUMBA cannot distinguish between (and may end up dying no matter what!), but you just need to program a VPR that solves as many of the scenarios as possible, while remaining as small as possible: your score depends on how small you can make your VPR. In the diagrams, RUMBA starts at **R**, it must get to location **G** and the obstacle is marked with the thick lines in the middle.

5.5 Scoring

We will simulate your VPR on every one of the 9 scenarios. If RUMBA reaches the goal in a scenario, your VPR is considered a "winner". The score you get for a scenario is computed as follows:

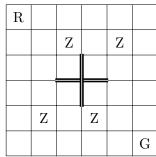
- If your VPR is not a "winner": 0
- If your VPR is a "winner": Size(VPR*)/Size(VPR), where Size(VPR) is the size of your VPR (the number of action nodes) and Size(VPR*) is the size of the smallest winner VPR from all of the teams.³

Note that the *best* score you can achieve for a scenario is 1.0, and you will only get points if you achieve the goal for the scenario. The best score for this question is 9, and so your score will be multiplied by 30 to get the final mark for this question.

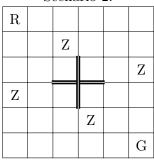
²Hint: this could be a good strategy!

³To illustrate, the size of the example VPR is 14.

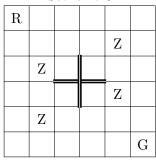
Scenario 1:



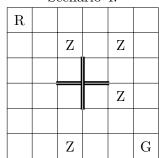
Scenario 2:



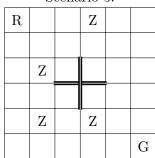
Scenario 3:



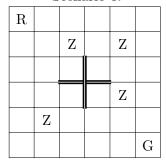
Scenario 4:



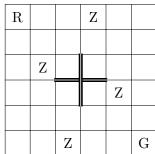
Scenario 5:



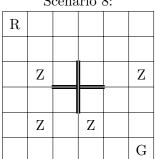
Scenario 6:



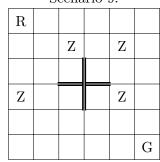
Scenario 7:



Scenario 8:



Scenario 9:



6 League of Legends [150 marks]

You are playing one of the characters, called TORCH, in a multi-player video game. TORCH tries to absorb as much damage from the enemy as possible to protect other players (thus, TORCH is a tank character).

6.1 Game Rules

A detailed description of the game rules:

- 1) TORCH enemies will attack TORCH in a combination of Physical Damage (PDmg), Magic Damage (MDmg) and True Damage (TDmg).
- 2) TORCH has a fixed amount of gold to purchase up to three equipment items that add to its health (HP), its physical resistance (PR), and its magic resistance (MR). Items available for purchase with their costs and bonus contributions to HP, PR, and MR are listed in Table 1.
- 3) The base statistics of TORCH's HP, PR, and MR are shown in the first row of Table 2.
- 4) PR reduces the physical damage that TORCH takes from PDmg according to the formula below:

Physical Damage Taken (PDT) = PDmg ×
$$\frac{100}{(100 + PR)}$$

5) MR reduces the magical damage that TORCH takes from MDmg according to the following formula:

$$\label{eq:magic_def} \text{Magic Damage Taken (MDT)} = \text{MDmg} \times \frac{100}{(100 + \text{MR})}$$

6) True damage ignores resistance, i.e.,

True Damage Taken
$$(TDT) = TDmg$$

- 7) The base HP being equal to 2000 represents the amount of damage TORCH can take before dying.
- 8) TORCH wants to withstand as much total damage (Dmg=PDmg+MDmg+TDmg) as possible while the total damage taken (PDT+MDT+TDT) is less than or equal to its HP.
- 9) After careful study of the enemies, you realize that each enemy attacks with constant proportion of PDmg, MDmg and TDmg denoted as (α, β, τ) where $\alpha + \beta + \tau = 1$.

$$\alpha = \frac{\text{PDmg}}{\text{Dmg}}, \beta = \frac{\text{MDmg}}{\text{Dmg}}, \tau = \frac{\text{TDmg}}{\text{Dmg}}.$$

6.2 Enemies

- a) [50 marks] CROTH, enemy of TORCH, will attack with 0% Physical Damage, 0% Magic Damage and 100% True Damage, (0, 0, 1). TORCH has 8000 gold to spend, and can purchase up to three equipment items to maximize the total damage that it can withstand. Assuming that you can buy at most one item of each equipment type, identify the equipment items and calculate the total damage.
- b) [50 marks] ROTCH, another enemy of TORCH, attacks with (0.6, 0.3, 0.1) proportion. TORCH has 8000 gold to spend, and can purchase up to three equipment items to maximize the total damage that it can withstand. Assuming that you can buy at most one item of each equipment type, identify the equipment items and calculate the total damage.
- c) [50 marks] After defeating CROTH and ROTCH, TORCH was able to purchase all six equipment items and its new statistics are shown in the second row of Table 2. You have changed your character in the game and are now playing HORCT, the arch-enemy of TORCH. HORCT will duel with TORCH but HORCT cannot attack with True Damage. Devise an attack plan for HORCT, $(\alpha, \beta, \tau = 0)$, to destroy TORCH with minimal damage inflicted by HORCT. In the other words, you want to minimize the total damage that HORCT inflicts, Dmg=PDmg+MDmg, such that the total damage that TORCH takes, PDT+MDT, is greater than its HP.

Equipment	Cost	Bonus HP	Bonus PR	Bonus MR
Warmog	2650	+1000	0	0
Veil	2500	+300	0	+45
Sunfire	2500	+450	+45	0
Bulwark	3200	+400	+20	+30
Thornmail	2200	0	+100	0
Randuin	3100	+500	+70	0

Table 1: Equipment available for purchase

TORCH	HP	PR	MR
Base	2000	0	0
Fully equipped	4650	235	75

Table 2: TORCH statistics

7 Radiotherapy Treatment [100 marks]

External beam radiotherapy is a common treatment of tumors. In this kind of treatment, a radiotherapy beam delivers a certain dose to tumor tissues to kill them. However, the surrounding healthy tissues are also damaged by the dose.

Previous studies show that the damage to the tumor and surrounding healthy tissues is dependent on the type of the beam, the dose delivered to the tumor and the tissue type. The challenge of this treatment is in determining the dose delivered by each type of beam to destroy as many tumor tissues as possible without causing too much damage to healthy surrounding tissues.

The Princess Margaret Hospital has recently contacted your research group to help them with devising a treatment plan for tumors in the lungs.

Lung tumors radiotherapy treatment There are four different radiotherapy beams for treating tumors in the lungs. Each beam kills a certain number of tumor and healthy tissues.

Figure 1 shows the position of the tumor and the surrounding healthy tissues. The gray region (in the centre) is the tumor, the top region (within the thick border) is the healthy lung tissues, the bottom left region within the thick border represents the healthy liver tissues, and the bottom right is the healthy gallbladder tissues. The four channels numbered from 1 to 4 represent the positions and the angles of the four beams. Once a beam is administered from one direction, it will go through all grid cells that are in its straight path. For example, if a beam from channel 1 is administered, the beam will hit 5 healthy cells in the lung, 2 tumor cells, and 1 healthy cell in the gallbladder.

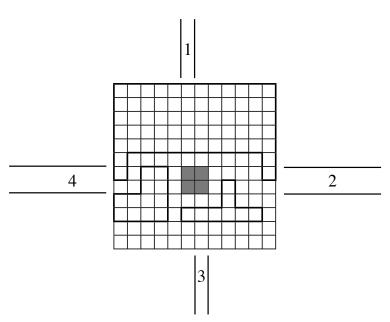


Figure 1: The position of tumor tissues, healthy tissues, and beams

The damage to different tissues using different beams (shown in Table 3) is dependent on the dose delivered and on the number of cells that a beam hits in its path, denoted as n. Remember, n can be different for different beams and different tumor and healthy tissues. In Table 3, LTT, LHT, VHT, and GHT, respectively, stand for lung tumor tissues, lung healthy tissues, liver healthy tissues, and gallbladder healthy tissues. For example, beam 1 with dose 1 destroys 20 percent of the tumor tissues, 27 percent of lung healthy tissues, 0 percent of liver healthy tissues, and 7 percent of gallbladder healthy tissues. Your job is to find all the possible combinations of the dose delivered by four different beams to maximize the percentage of killed tumor tissues while observing the following three requirements by physicians:

- 1) At most 40 percent of the liver cells can be destroyed.
- 2) At most 20 percent of the lung cells can be destroyed.
- 3) At most 35 percent of the gallbladder cells can be destroyed.

Note that the dose delivered by beams will not result in more than 100 percent damage to tumor and healthy tissues. For example, beam 1 with does 2 kills 100 percent of lung healthy tissues.

The format of your solution should be (d_1, d_2, d_3, d_4) where d_i is the amount of dose delivered by beam i. Note that d_i can be any number greater than 0 and does not need to be an integer.

Beam type	Percentage of damage to different tissues			
	LTT	LHT	VHT	GHT
Beam 1	$10 \times n \times d_1$	$(5 \times n + 2) \times d_1^2$	0	$(6 \times n + 1) \times d_1$
Beam 2	$30 \times n \times d_2$	$(5 \times n + 0.5) \times d_2^2$	$2 \times n \times d_2$	$3 \times n \times d_2$
Beam 3	$20 \times n \times d_3$	$(5 \times n + 2) \times d_3^2$	0	$(6 \times n + 0.01) \times d_3$
Beam 4	$10 \times n \times d_4$	$(5 \times n + 0.5) \times d_4$	$2 \times n \times d_4$	$(3 \times n + 1) \times d_4$

Table 3: Percentage of damage to different tissues by different beams

8 Prime and Unique

Select a prime number between 2 and 40 (inclusive). The selected prime number will be the number of marks awarded to your group if no other group selects the same prime number. If two or more groups select the same prime number, all of these groups will receive no marks. What will your prime and unique choice be?

Attention: Write you answer below and submit this page with the answer sheet. Do not forget to write your group number.

Your Prime and Unique number is