Instructions

- This is Part I of the competition. There are 4 questions and it is 15 pages long. Please confirm that you have the proper booklet and that no pages are missing.
- You have 2 hours and 15 minutes to write this part of the competition.
- Please first write your group number on each page of this booklet. Note that failure to do so may result in your answers getting lost and your group receiving no marks for the corresponding questions.
- For each question, there is space in this booklet for writing your answer. Please write your final solutions in the indicated places, in the format explained in the questions. Answers stated on any other papers will NOT be marked.
- Please submit only the answer sheets that you want marked.
- Answers that are infeasible will be heavily penalized. Please ensure that all your answers satisfy the constraints given in each of the questions.

1 Noah's Nautical Dilemma [150 marks]

Background & Task

Noah's boss is a busy man. He oversees every zoo in North America. Following a particularly severe avian flu outbreak in several of his zoos, he assigns his model employee, Noah, the task of building a cargo ship to contain some animals and protect them from the avian flu, which is contagious between species (though luckily not humans).

Nearby, there is a cargo ship, depicted in Figure 1.1, containing 18 animal pens. *In order to reduce the risk of losing all of the animals to the growing pandemic, Noah's boss has requested that a finite number of the animals be loaded onto the cargo ship and transported to a temporary holding area until the flu is contained.* There is, however, only sufficient time to move one batch of animals before the risk of transporting already infected animals grows, becoming prohibitively large and it is up to Noah to decide how to stock the ship.

Constraints

Noah promptly compiles a list of the zoo's 15 most popular species of animals, transcribing on a per-animal-basis, the space it occupies, the creature's monetary value, weight, the quantity of that species possessed by the zoo, as well as the probability that the animal has already been exposed to the flu virus, denoted as Prob(Infect), which is a number between 0 and 1. As an afterthought, he adds a column for forbidden interactions, to ensure he does not place prey next to predators in case it instigates a breach between adjacent pens. His list is displayed in Table 1.1.

The layout of the pens is depicted in Figure 1.2 and each pen has an area and adjacency described in Table 1.2.

Additional Assumptions & Parameters:

1. The probability of an infection spreading is additive, meaning that each additional animal's Prob(Infect) is added to the group's combined probability of infection (which starts at 0).

2. If one infected animal is brought onboard, the close quarters will cause the entire ship to be compromised (driving the expected value of the entire ship to \$0).

3. Noah would like to maximize the total expected value of the Cargo ship from a financial perspective. The expected value can be calculated as (using the above assumptions):

$$\text{Expected Value} = \text{Total \$ Value of All Animals} \times (1 - \text{Sum of Prob(Infect)}) \tag{1}$$

4. The stairs on the left connect the bottom and middle floor, stairs on the right connect the middle and top floors.

5. An animal may be enticed to break out in order to climb/descend the stairs, or to enter pens that are connected on a diagonal, so pens connected by a staircase are considered adjacent, if they are between floors, i.e. pen 5 and 9 are adjacent.

6. Animals on the same floor are not incentivized to cross a staircase to get to another pen, i.e. pens 6 and 10 are not adjacent.

7. Each individual animal on the top floor must weigh less than 251kg and each individual animal on the middle floor must weigh less than 401kg. There is no weight restrictions for the bottom floor.

8. Different species of animals may NOT share the same pen, but a single species may occupy multiple pens.
9. Each individual animal has a needed area (second column in Table 1.1), and for each pen, the total needed area for all the animals is additive (i.e. the needed area of two animals is the sum of the needed area of each individual animal). The total needed area must not exceed the area of the pen.
10. Pen sizes are not to be changed and there are no fractional animals.
11. Note that there are only a limited number of each animal that can be taken into the ship, given in the 'Quantity' column of Table 1.1.

Question:

Which animals should Noah assign to each pen?

Tables & Figures:

Figure 1.1: Noah's cargo ship, retrofitted for animal transport

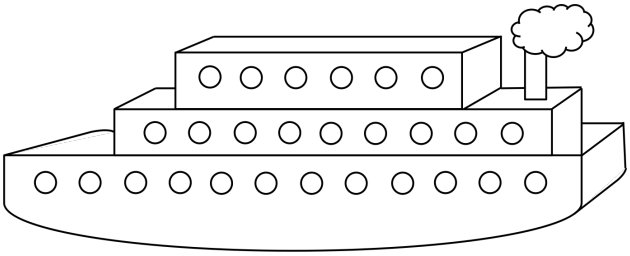
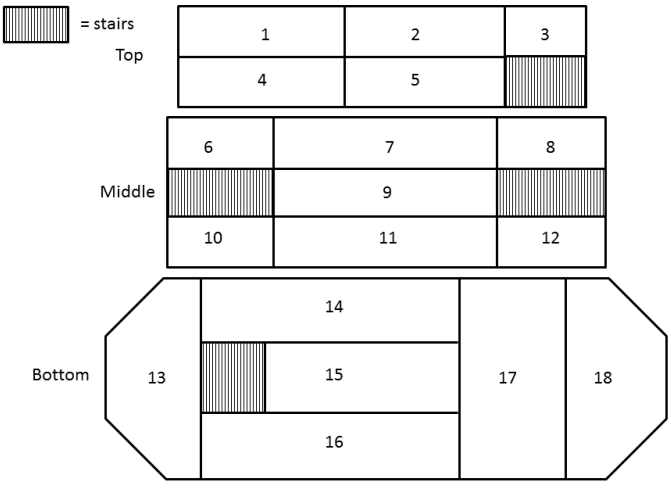


Figure 1.2: Rough layout of the cargo ship pens; pens 1-5 are on the top floor, 6-12 on the middle, and 13-18 on the bottom floor.



* Passageways between pens not depicted

Table 1.1: Animal species and their characteristics. For a given row, species listed in the Predators column are the predators of that row's animal, e.g. Eagles are prey to Lions, Tigers, Bears, Crocodiles and Komodo dragons.

#	Species	Area (ft^2)	Value ($)	Weight (kg)	Quantity	Prob(Infect)	Predators
1	Lions	50	1,000,000	200	11	.004	-
2	Tigers	50	1,250,000	250	3	.005	-
3	Bears	100	1,000,000	400	6	.002	-
4	Elephants	500	500,000	5,500	7	.001	-
5	Snakes	20	10,000	50	14	.007	-
6	Eagles	30	50,000	20	5	.025	1,2,3,10,11
7	Deer	40	5,000	5	26	.002	1,2,3,5,6,10,11
8	Zebras	75	75,000	500	9	.003	1,2,3,6,10,11
9	Hippos	300	1,000,000	2,000	4	.005	-
10	Crocodiles	70	500,000	100	8	.009	-
11	Komodo dragons	50	200,000	120	3	.010	-
12	Kangaroos	60	400,000	250	6	.007	1,2,10
13	Camels	400	750,000	1,500	2	.003	1,2,3,10,11
14	Pandas	100	2,000,000	600	4	.004	10
15	Monkeys	40	60,000	80	15	.001	1,2,3,5,6,10,11

Table 1.2: Pen description

Pen	Area (ft^2)	Adjacency	**Pen**	Area (ft^2)	Adjacency
1	100	2,4,5	10	150	9,11,13,14,15,16
2	100	1,3,4,5,7,8,9,11,12	11	300	2,3,5,9,10,12,13,14,15,16
3	50	2,5,7,8,9,11,12	12	150	2,3,5,9,11
4	100	1,2,5	13	500	6,7,9,10,11,14,16
5	100	1,2,3,4,7,8,9,11,12	14	500	6,7,9,10,11,13,15,17
6	150	7,9,13,14,15,16	15	400	6,7,9,10,11,14,16,17
7	300	2,3,5,6,8,9,13,14,15,16	16	500	6,7,9,10,11,13,15,17
8	150	2,3,5,7,9	17	500	14,15,16,18
9	300	2,3,5,6,7,8,10,11,12,13,14,15,16	18	500	17

Example answer:

Consider the following example using Deer (7) and Hippos (9):

Pen	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Species #	7	7	7	7	7	7	7		7						9	9	9	9
Quantity	2	2	1	2	2	3	7		7						1	1	1	1

The total expected value is: To calculate the total expected value, we first need to calculate the value of all the animals on board, which is: $\$5,000 \times 26 + \$1,000,000 \times 4 = \$4,130,000$. Then, we need to find the total Prob(Infect), which is: $0.002 \times 26 + 0.005 \times 4 = 0.072$. Then the total expected value is $\$4,130,000 \times (1 - 0.072) + \$0 \times 0.072 = \$3,832,640$.

Note that not all the pens have to be filled.

Answer:

Fill in the species number and quantity of the selected species assigned to each pen.

Pen	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Species #																		
Quantity																		

The total expected value is: $\$$_____

2 Wheat Transportation [100 marks]

Background & Task

As manager of an eastern wheat cooperative you have to arrange the delivery of an order for 600 tons of wheat, stored among 4 silo locations, to arrive at a port in Toronto no later than 25 days from today (This question was partly inspired by a problem set by Daniel Frances, University of Toronto.)

Constraints

Each silo has a limited amount of wheat available, given in Table 2.1. You will be ready to transport the wheat from the silos starting the morning of day 1, first by rail to one of 4 ports on the Great Lakes, and from there by ship. Figure 2.1 shows the different possible paths starting from the silos, through the ports and finally ending at Toronto. Within the next 25 days, there are several trains scheduled to travel from each silo to each port; and there are 2 ships sailing for Toronto from each of the 4 ports. See Tables 2.2 and 2.3 for rail and ship schedules (assume that today is day 0, and the wheat is due in Toronto on day 25). Trains arrive at each port early in the morning, and ships depart late in the day, providing sufficient time to load the ships. Each ship also has a limited capacity for wheat, listed in Table 2.3.

Additional Assumptions & Parameters:

1. There is a fixed charge of $50,000 for using each train, in addition to a variable cost of $100/ton of wheat transported by rail.
2. Similarly, there is a fixed charge of $200,000 for using each ship, in addition to a variable cost of $150/ton.
3. There is a storage cost of $300/ton/day for wheat that sits at a port before departure by ship.
4. Finally, there is an incentive of $400/ton/day for wheat that reaches Toronto early.

Question:

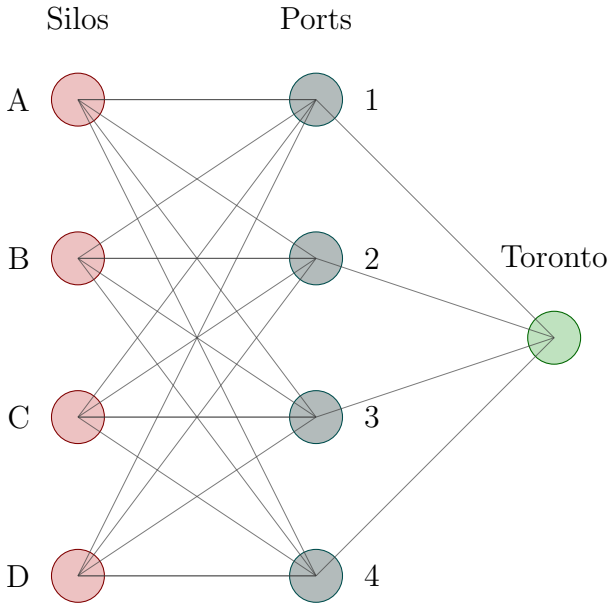
Find the most inexpensive transportation plan that meets the deadline, and enter your answers in the rightmost columns of Tables 2.2 and 2.3. The number you put in the rightmost column represents the tons of wheat you wish to move along that mode of transport. For example, putting 100 beside the first row in Table 2.2 means that 100 tons of wheat will be transported from Silo A to Port 1 via a train that leaves on Day 3 and arrives on Day 12. Then 100 tons of wheat are put into Port 1. Note that you *do not* have to transport all the wheat in each silo, you only have to transport 600 out of the 800 tons of wheat.

Tables & Figures:

Table 2.1: Wheat Available at Each Silo

Silo	Wheat Available
A	200
B	250
C	50
D	300

Figure 2.1: Silo to port map



Answer:

Table 2.2: Schedule of trains traveling from each silo to each port. Use the rightmost column to enter your answers.

Silo of Departure	Port of Arrival	Departure Day	Arrival Day	Tons of wheat to transport (your solution)
A	1	3	12	
A	1	13	22	
A	2	8	11	
A	2	14	17	
A	3	5	21	
A	4	6	20	
B	1	10	17	
B	2	12	15	
B	2	13	16	
B	3	8	16	
B	4	3	10	
B	4	15	22	
C	1	11	16	
C	2	12	20	
C	2	4	12	
C	3	4	19	
C	3	2	17	
C	4	8	22	
D	1	10	17	
D	2	6	20	
D	3	11	18	
D	4	6	10	
D	4	13	17	

Table 2.3: Schedule and capacity of ships traveling from each port to Toronto. Use the rightmost column to enter your answers.

Port of Departure	Departure Day	Day of Arrival at Toronto	Ship Capacity Tons	Tons of wheat to transport (your solution)
1	10	15	100	
1	20	23	100	
2	13	16	50	
2	22	25	50	
3	17	25	300	
3	18	24	200	
4	16	21	250	
4	19	22	100	

3 Solar Panel Allocation [150 marks]

Background & Task

Your engineering company has been hired to construct a solar panel farm to power a small neighborhood. This solar panel farm contains 10 solar panels, which have angles that can be independently fixed. However, once the angle is fixed, they can never be changed. Since all the houses in this neighborhood are 'smart homes', your solar panel farm is able to allocate energy that is generated into different appliances in all the different homes. *Your job in this question is to (1) decide the tilt angle of the solar panels so as to maximize energy absorption and (2) come up with an allocation of the energy generated into the different appliances in all the homes.*

Constraints

Solar Panels

The tilt angle of each solar panel affects how much energy it can produce in a time of day, depending on its angle against the angle of the sun at that time interval (we choose two hours as the unit of measure resulting in 12 two-hour intervals in a day). We exclude the times from 2200 to 0400 because there is no sun at this time. The angle of the sun for a given time of the day is given in Table 3.1.

The amount of energy that a solar panel can generate is a function of the difference of the set angle of the solar panel and the angle of the sun at a given time. Let θ denote the angle of a solar panel, where $0^\circ \leq \theta \leq 90^\circ$. Let α denote the angle of the Sun. Then the total energy (per solar panel) generated in a two hour window (in 'energy units') is:

$$E = \begin{cases} 100 - |\alpha - \theta| & \text{if } \alpha > 0 \\ 0 & \text{if } \alpha = 0. \end{cases} \tag{2}$$

Energy Requirements

The small neighborhood has a set of homes whose energy demands are pooled together, based on the different appliances: the electric car, AC/Heat, lights, and kitchen. The energy requirements change over the course of the day. The number of energy units required for each of the demands is given in Table 3.2. You have to, over the course of the entire day, satisfy the 'Required Total', listed as 'Req. Total' on the table. Note that there is a minimum amount of energy that you are required to allocate to the different appliances, and at certain times you are not allowed to allocate any energy. These are given in Table 3.2.

Battery & Electric Grid

Your neighborhood also installed a battery that can store energy throughout the day. The total capacity of the battery is limited to 1000 energy units. Any energy generated by the solar panels and not used/assigned by any appliances is immediately transferred to the battery. However, since the battery has limited capacity, any solar energy that is transferred to the battery when it is saturated will be considered as 'overflow' to the main electric grid. This causes instability and you will be charged \$3 per energy unit of overflow.

There are some cases when you may choose to not use battery energy, or you are not able to because the battery is empty. In this case, you may draw energy from the grid to meet demand at a cost of \$3 per energy unit used. You may not use the main electric grid to charge the battery.

Additional Assumptions & Parameters:

1. The solar panels do not all have to be set to the same angle.
2. Any solar energy that is not used by the appliances is automatically transferred to the battery.
3. If your neighborhood has a shortage of power (either because the solar energy is insufficient or you cannot/choose not to use the battery), it must take energy from the main grid. The cost of using energy from the grid is \$3 - the same for any appliance.
4. The battery does *not* always have to be used when there is a shortage of energy, you have the option of drawing from the grid instead.
5. The battery has a maximum capacity of 1000 energy units. Once the battery is filled to capacity, it overflows, and the extra energy units causes instability in the main grid. This instability is a cost to the neighborhood, and it has to pay an instability fee depending on how much energy overflowed. Overflow from the battery costs \$3 per energy unit.
6. Assume that the battery is always re-charged at the beginning of the day up to capacity by an external nuclear reactor, so that at 0400, the battery is fully charged to 1000 energy units.

Question:

Find the appropriate angles of the 10 solar panels, and the allocation of the total generated energy to the different appliances at particular times of the day. That is, you must decide when the energy is distributed to the appliances, so long as they satisfy the minimal demands and required totals given in Table 3.2. Find the allocation that minimizes the total cost to the neighborhood.

Tables & Figures:

Table 3.1: Angles of the Sun at different times of the day

	AM				PM					
Time	04	06	08	10	12	14	16	18	20	22
Angle: (in °)	0	5	25	45	65	65	45	25	5	0

Table 3.2: Minimum required energy for different times of the day, and the total required energy that has to be distributed over the entire day. The '×' means no energy may be allocated to that appliance at that time.

		AM				PM					
Time		04	06	08	10	12	14	16	18	20	22
	Req. Total	Minimum									
Electric Car	2000	300	100	50	×	×	×	×	50	100	300
AC/Heat	1600	300	50	0	×	0	0	×	0	50	300
Lights	2000	300	50	0	0	×	×	0	0	50	300
Kitchen	600	0	100	50	0	0	0	0	50	100	0

Example Answer:

Consider the case when we select the same angle for all the solar panels (they don't all have to be the same angle, but just for the sake of example). This is the naïve version

Table 3.3: **Example answer:** Panel Angles

Panel	1	2	3	4	5	6	7	8	9	10
Angle	45	45	45	45	45	45	45	45	45	45

Table 3.4: **Example Answer:** Allocation of Energy.

	AM				PM					
Time	04	06	08	10	12	14	16	18	20	22
Solar Energy	0	600	800	1000	800	800	1000	800	600	0
Battery Used	1000	0	0	0	0	0	0	100	100	800
From Grid	0	100	100	0	0	0	0	0	0	200
Electric Car	300	200	500	×	×	×	×	500	200	300
AC/Heat	300	300	200	×	0	0	×	200	300	300
Lights	400	100	0	500	×	×	500	0	100	400
Kitchen	0	100	200	0	0	0	0	200	100	0
Overflow	0	0	0	0	300	800	500	0	0	0
Battery Charge	0	0	0	500	1000	1000	1000	900	800	0

Note that for any given time period, the total energy used amongst the appliances must always add up to a number less than the total energy supplied by the 'Solar Energy', 'Battery Used' and 'From Grid' section. The total energy allocated to each of the appliances satisfies the required demand. For example, the sum of the total energy allocated to the electric car over all time is exactly 2000 energy units. At 10 AM, 1000 energy units are generated with 500 dedicated to the Lights and the remaining 500 stored in the battery. Furthermore, at 12 noon, 800 units of energy are generated, 500 of which are allocated to the battery remaining, and the rest overflows to the main grid. As can be seen in Table 3.4, the total cost to the neighborhood is $(100+100+200) \times \$3$ from using energy from the grid, and $(300 + 800 + 500) \times \$3$ from the overflow. This leads to a total cost of \$6000.

Answer:

Give your answer in Tables 3.5 and 3.6.

Table 3.5: **Answer:** Panel Angles

Panel	1	2	3	4	5	6	7	8	9	10
Angle										

Table 3.6: **Your Answer:** Allocation of Energy

	AM				PM					
Time	04	06	08	10	12	14	16	18	20	22
Solar Energy	0									0
Battery Used										
From Grid:										
Electric Car				×	×	×	×			
AC/Heat				×			×			
Lights					×	×				
Kitchen										
Battery Charge										
Overflow										

Total cost in $: _____

4 Multidisciplinary Study [100 marks]

Background & Task

In this question, you will assemble a sub-team of researchers to participate in a multi-disciplinary study to be conducted at the University of Toronto.

Constraints

The sub-team that you choose will then be grouped together with a sub-team selected by another TORCH group's sub-team, to form a full-team. You are given a funding package of $1,500,000 for your own sub-team (i.e. not the combined full team), for which you may choose an assortment of personnel, whose cost and raw research ability are given in Table 4.1.

Your goal will be to maximize your sub-team's research ability. However, besides the raw research ability, there are added bonuses or penalties for certain configurations of people on the combined full-team. These bonuses or penalties result from having different types of researchers influencing the full-team's dynamics. The bonuses and penalties are given in Table 4.2. Note that the bonuses are independent, so that 2 Nobels may participate both in an 'Optimal Nobel' as well as a 'Royal Flush'. Also note that the bonus is *not* always evenly distributed amongst the two sub-teams, and that one sub-team may receive a bonus while another may receive a penalty.

Further clarification on Table 4.2: Consider the 'Balanced Team' bonus. The bonus of +2000 to Nobel and +1000 to Alpha imply that +2000 is given to the sub-team with the Nobel, while only +1000 is given to the sub-team with Alpha, or, +3000 to the sub-team that has both. The 'Full Team Composition' column means that your full team must satisfy the condition listed. You are still allowed to have other group members there as long as it does not violate the composition requirement. As another example, consider the 'Full Lab' bonus. While the bonus is awarded if the combined full group has at least 20 Initiates and 10 Novices, the points are allotted only according to the distribution among the sub-teams. One potential scenario with uneven point distributions is: one sub-team donates 15 Initiates and receives 15×250 points; the other sub-team donates 5 Initiates and 10 Novices and gets 5×250 + 10×500. So the points distributions are not even.

Question:

Choose a sub-team that does not exceed the provided funds of $1,500,000 and maximizes your sub-team's research ability.

Tables & Figures:

Table 4.1: Raw abilities of researchers

Title	Nobel	Alpha	Senior	Junior	Novice	Initiate
Ability	1500	800	375	150	60	25
Cost	$ 750,000	$500,000	$250,000	$100,000	$50,000	$25,000

Table 4.2: Combined abilities: For the Full Lab, bonuses apply to both sub-teams if both have sufficient Initiates or Novices. That is, up to 25 Initiates or 15 Novices per sub-team. For Kindergarten, the −12000 penalty applies only once, that is, even if at least 30 Initiates *and* 20 Novices are in the full team, only −12000 is applied to both teams.

Title	**Full Team Composition**	**Bonus/Penalty**
Royal Flush	At least 1 of every title	+10000 to both
High Rule	Exactly 2 Nobel & No Alphas	+4000 to both
Balanced Team	Exactly 1 Nobel & exactly 1 Alpha	+2000 to Nobel +1000 to Alpha
Full Lab	At least 20 Initiates & at least 10 Novices	+250 per Initiate (bonus only up to 25) +500 per Novice (bonus only up to 15)
Young Department	At least 3 Juniors At least 10 Novices	+2500 to both
Apprenticeship	Exactly 2 Seniors & at least 2 Novices	+5000 to both
Intimidation	One sub-team has exactly 2 Alphas & the other has none	+10000 to sub-team with Alphas −7500 to sub-team w/o Alphas
Anti-Nobel	Exactly 2 Nobels & and exactly 2 Alphas	−3000 to both
Imbalance	No Seniors & no Juniors	−3000 to both
Aging	Less than 10 Initiates	$-1000 \times (10 - \#\text{Initiates})$ to both
Kindergarten	At least 30 Initiates or at least 20 Novices	−12000 to both (applies once)

Example Answer:

Suppose you chose the following sub-team configuration, given in Table 4.3, and you were randomly assigned another TORCH group's sub-team configuration, given in Table 4.4.

Table 4.3: **Example Answer:** Your own sub-team configuration (Team 1)

Title	Nobel	Alpha	Senior	Junior	Novice	Initiate
Quantity	1	1	1	0	0	0

Table 4.4: **Example Answer:** Another group's sub-team configuration (Team 2)

Title	Nobel	Alpha	Senior	Junior	Novice	Initiate
Quantity	0	0	1	2	11	20

Note that the Royal Flush, Balanced Team, Full Lab and Apprenticeship bonuses are in effect, but the bonuses are not the same for Team 1 and Team 2. Team 1 is the only one that benefits from 'Balanced Team'. In the end, Team 1 has 20,675 points, and Team 2 has 27,335 points.

Answer:

Title	Nobel	Alpha	Senior	Junior	Novice	Initiate
Quantity						

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1 Dependence Day [100 marks]

Background & Task

The worst has happened. The aliens have landed with the intention of making the earth their own. Using their nefarious shape-shifting abilities, they have taken ranks among the most powerful government officials in order to assassinate the president. If they are not stopped, the rest of humanity will surely follow. The minister of defence has called an emergency meeting, where she outlines a plan that may be humanity's final shot at survival.

Deep in the classified government vaults, she explains, lies humanity's final shot at survival; a deadly weapon with a built-in detection system, the FaceBlaster500$^{\text{TM}}$. Unfortunately, due to a high potential for tampering, officials had feared that the weapon could be turned against humans, and it was classified as "highly dangerous" and locked down in a heavily booby-trapped vault. The minister of defence then provides some even more unsettling news; the only person in the world with blueprints to the vault was the late president and since his death, they have gone missing and are presumed lost forever. Without them, it would be near impossible to break in. *With a sudden chill, you realize why you have been asked to attend this meeting – you are told that if you can successfully break into the vault and retrieve the FaceBlaster500^{TM} you will be granted your freedom.*

Constraints

In a private chamber, the minister introduces you to three individuals Person A, Person B and Person C. The minister tells you that these are the three people that have helped architect the vault and your best chance at survival. Her voice wavers as she calls them "trustworthy" experts. She tells you that there are several known facts about the vault:

1. To bypass the sensors, your weight, including equipment must not exceed 100kg.
2. There is an override switch for the vault's security located directly above the FaceBlaster500$^{\text{TM}}$.
3. If you spend more than 300 seconds in the vault without overriding the security system, the door will lock from the inside and deadly chemicals will be released into the room (i.e. you cannot spend more than 300 seconds in the vault).
4. The room contains five deadly obstacles, chosen by the late president from a list of 10 possibilities listed in Table 1.1.

Person A then chimes in and says he remembers the vault having a poison lever, welded bars and a combination lock. Person B thoughtfully adds that he recalls trip wires, motion sensors and he agrees that welded bars were among the obstacles. While they talk, Person C hesitates. Finally he chimes in and says he does not recall welded bars being present, and insists that when he signed off on the document, attack dogs, trip lasers and high voltage items were included in the final blueprints.

Additional Assumptions & Parameters:

1. You have enough of whichever tools you choose to bring that you may use them on separate obstacles.
2. None of your tools get destroyed or lost.

3. Tools may not be combined for an advantage on a single obstacle.

4. Unless otherwise stated, assume that everyone is truly human.

You nod, frowning at the lack of consensus. The minister avoids your questioning glance. You step on a scale with an empty pack and find you weigh 65kg. You then glance at the table and evaluate your tool choices, described in Table 1.2. Upon careful consideration, you choose a subset and dump them unceremoniously in your pack.

Tables & Figures:

Table 1.1: Potential Obstacles

#	Obstacle	Estimated Time to Disable By Hand (sec)
1	Attack Dogs	50
2	Poison Lever	60
3	Trip Lasers	100
4	Trip Wires	40
5	Sound Vibrations	70
6	High Voltage	120
7	Motion Sensors	130
8	Airborne Poisons	110
9	Welded Bars	180
10	Combination Lock	25

Table 1.2: Description of tools, '-' indicates a non-useful tool for that item

Tool Description			**Time to Disable Obstacle with Tool (sec)**									
#	Tool	Weight (kg)	1	2	3	4	5	6	7	8	9	10
1	Gas Mask	9	-	50	-	-	-	-	-	20	-	-
2	Industrial Earplugs	11	-	-	-	-	15	-	-	-	-	-
3	Stethoscope	4	-	-	-	-	-	-	-	-	-	8
4	High Voltage Gloves	12	-	20	-	-	-	35	-	-	-	-
5	Throwing Stars	5	-	-	-	15	-	-	90	-	-	-
6	Aerosal Canistor	7	-	-	25	-	-	-	-	-	-	-
7	Blow Torch	15	-	-	-	-	-	-	-	-	55	-
8	Smoke Bombs	13	50	-	80	-	-	-	20	-	-	-
9	Laced Meat	8	10	-	-	-	-	-	-	-	-	-

Questions & Answers:

a) Assuming you distrust all three of the people and treat all 10 obstacles with equal likelihood, which tools do you choose? (That is, each potential obstacle has exactly the same probability of occurring.)

Tools: _____
How long do you expect your mission to take? _____

b) Assuming you distrust Person C, but trust A and B, which tools do you choose? (That is, ignore anything that Person C says, and take what Person A and B say as absolute truth.)

Tools: _____
How long do you expect your mission to take? _____
What if you used your tools from part a)? How long would you expect it to take? _____

c) Assuming Person A is a shape-shifting alien that is trying to mislead you and Person B and C do not actually know what they are talking about, which tools do you choose? (That is, Person A's obstacles of 2, 9, & 10 are absolutely not in the vault; and you can ignore the statements made by Person B or Person C by assuming that the remaining (i.e. other than 2, 9, & 10) obstacles are equally likely to occur.)

Tools: _____
How long do you expect your mission to take? _____
What if you used your tools from part b)? How long would you expect it to take? _____

d) What if you suspect Person A and Person B are shape shifting aliens and Person C is truly your ally? (That is, Person A and B's obstacles are absolutely not in the vault, and Person C's obstacles are absolutely in the vault.)

Tools: _____
How long do you expect your mission to take? _____
What if you used your tools from part c)? How long would you expect it to take? _____

2 Fleet of Xandor II - Assault on the Colonies [150 marks]

Background & Task

There has been a devastating assault on the colony worlds of the Xandorian Republic. While the enemies have been destroyed, the colonies are in desperate need of supplies. Each of the six colonies require supplies unique to their environment. The required supply types are labeled A to F. The Xandorian Relief Fleet has six different classes of ships, each designed to carry one, and only one, type of supply. They are labeled by the supply type it can carry. *Your task is to find a set of upgrades for your ships that allows you to satisfy as much of the as-of-yet unknown demand in the different colonies as possible.*

Constraints

We do not have perfect information about the demand for relief supplies in each of the colonies, but we do know that they are one of 15 scenarios given in Table 2.1. The units in the table are in kT of cargo. Each of these scenarios is equally possible, but there will be no further information until one week from now.

In the meantime, the Fleet Admirals have authorized upgrading the ships in the fleet to carry more than one type of cargo, so we can respond better to the different demands at each colony when they come out. Each ship can carry exactly 1 kiloton (kT) of cargo, and there are 100 ships of each class in the Fleet. The current network showing which ships can bring supply to which colonies is given in Fig. 2.1. Given our current fleet without upgrades, if say, Scenario 1 occurs, then Colonies A and B will have a shortage of 50 kT each, a total of 100 kT of shortage. However, consider the example given in Table 2.3. If Scenario 1 occurs, and if we had upgraded the class C ship to also take care of Colony B as in Fig. 2.2, we could transfer 25 kT to Colony B, reducing the shortage down to a total of 75 kT. The network given in Fig. 2.1 can be represented as in Table 2.2. However, because class D ships were not upgraded to handle cargo bound for Colony A or B, none of its extra capacity can be used to supply Colonies A or B.

Additional Assumptions & Parameters:

1. Each kiloton of shortage costs 1 credit.
2. All the ships originate from the same planet, and that this planet is full of factories that can produce anything we want, so there can be no shortage of supply.
3. Ships can only be used once. They do not make round trips or go back to refill once they have dropped off their supply once at a colony.
4. Each individual ship can only carry 1 kiloton of any particular type of supply. So a ship cannot carry 0.5 kT of type A and 0.5 kT of type B.
5. As a clarification, the upgrades work by allowing ships to transport materials to colonies of a different type. So in our example above, the upgrade to the class C ship allowed it to transport some material to Colony B.

Question:

You must choose a particular network configuration of upgrades that cannot change no matter which scenario appears. For each possible scenario, there is a *minimal* cost that can be achieved by

correctly assigning upgraded ships to colonies. Having the wrong upgrades means that you cannot send ships to where they ought to go and thus may incur a high cost for a given scenario.

Select upgrades for the ships to minimize the average *minimal* costs over all the scenarios, given that you get 6 free upgrades, and each additional upgrade costs 15 credits.

Tables & Figures:

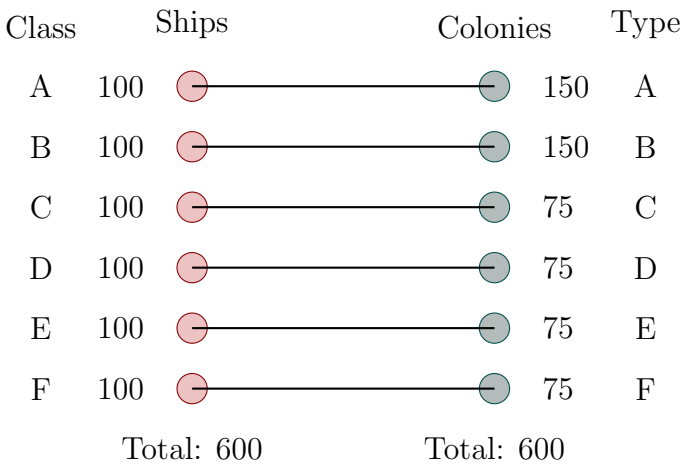
Table 2.1: 15 different possible demands of equal probability. The units here are in kT of cargo.

		Scenarios														
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Colonies	A	150	150	150	150	150	75	75	75	75	75	75	75	75	75	75
	B	150	75	75	75	75	150	150	150	150	75	75	75	75	75	75
	C	75	150	75	75	75	150	75	75	75	150	150	150	75	75	75
	D	75	75	150	75	75	75	150	75	75	150	75	75	150	150	75
	E	75	75	75	150	75	75	75	150	75	75	150	75	150	75	150
	F	75	75	75	75	150	75	75	75	150	75	75	150	75	150	150

Table 2.2: Default configuration, with 0 for no upgrade, and 1 for existing or added upgrade

		Colonies					
		A	B	C	D	E	F
Ships	A	1	0	0	0	0	0
	B	0	1	0	0	0	0
	C	0	0	1	0	0	0
	D	0	0	0	1	0	0
	E	0	0	0	0	1	0
	F	0	0	0	0	0	1

Figure 2.1: Initial Fleet Configuration, under Scenario 1



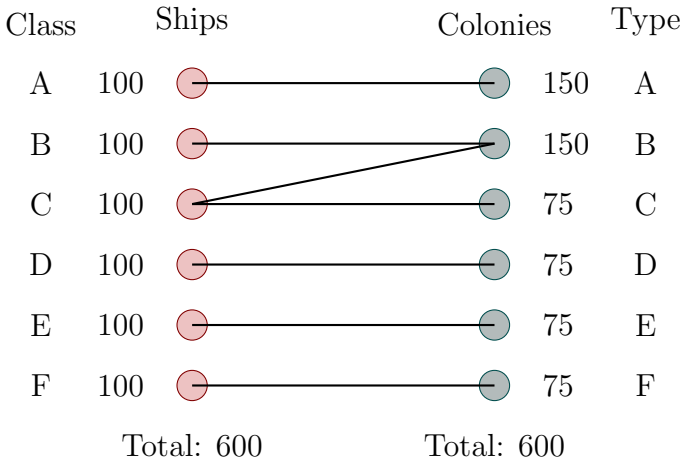
Example Answer:

Upgrading the class C ship will lead to Figure 2.2, and Table 2.3.

Table 2.3: Example upgrades

		Colonies					
		A	B	C	D	E	F
Ships	A	1	0	0	0	0	0
	B	0	1	0	0	0	0
	C	0	1	1	0	0	0
	D	0	0	0	1	0	0
	E	0	0	0	0	1	0
	F	0	0	0	0	0	1

Figure 2.2: C-upgraded Fleet Configuration, under Scenario 1



Thus, the upgrades limit how you are able to deliver supplies to the colonies. Given a fixed upgrade configuration, you will either be able to satisfy the demand for a given scenario, or you will incur some demand shortage. In Fig. 2.2, we find that the total shortage is 75. For a different scenario, say, scenario 2, the total shortage is 100. Note that you will have to get the average over all the scenarios. To obtain this value, we need to check, for each different Scenario, how to satisfy the demand with the network configuration given in Fig. 2.2. One can check that the costs in Table 2.4:

Table 2.4: 15 different costs based on the Scenarios. Note there is no incurred cost for adding the link from C to B, because 6 free upgrades are included.

Scenario	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Costs	75	100	100	100	100	100	75	75	75	100	100	100	100	100	100

In this case, the total average cost (over the 15 scenarios) is $93.\overline{3}$ credits.

Answer:

Table 2.5: **Answer:** Give your answer here - leave blank or put 0 for no upgrade and put 1 for an upgrade

		Colonies					
		A	B	C	D	E	F
Ships	A	1					
	B		1				
	C			1			
	D				1		
	E					1	
	F						1

Total cost in credits: _____

3 Traders of the Pacific - 2106 AD [100 marks]

Background & Task

In the year 2106, you are aboard the trading ship *Fiery Beacon* travelling between the eight main trading ports of the Pacific: Manila, Tokyo, Bering, Anchorage, Vancouver, Santiago, Sydney, and Antarctica, shown in Figure 3.1. *In this question, your task is to determine the trade routes that earn the most money. You make profit by purchasing goods at a lower price in one city, and selling them at a higher price in another city.*

Constraints

Each trading port has some subset of 9 types of goods, with a single buying/selling price for each good. These are given in Table 3.1. Note that not all cities trade all goods.

Additional Assumptions & Parameters:

1. You always start off with ample capital to buy anything at any port. However, the goal of this question is to find a trade route that maximizes the total profit.
2. The ship has a capacity of 200 kilotons.
3. You begin and end in Manila, where you may buy and sell goods, but you are not required to.
4. Assume that you always have enough money to purchase any good, up to the ship's capacity.
5. The ship may travel to the same city twice.
6. You may not 'idle' at a city.
7. You may both buy and sell at a city, as long as you do not exceed the ship's capacity.
8. You cannot purchase in partial kilotons.
9. You *may* mix your cargo. So you can buy 50 kilotons of Food and 100 kilotons of Machines.
10. You may carry cargo for as long as you like.
11. The ship can only travel in straight lines in Figure 3.1. For example, you may not travel directly from Manila to Anchorage.

Question:

Beginning in Manila, find the profit-maximizing trade route that is exactly 6 links long and also ends in Manila.

Tables & Figures:

Figure 3.1: Eight main trading ports

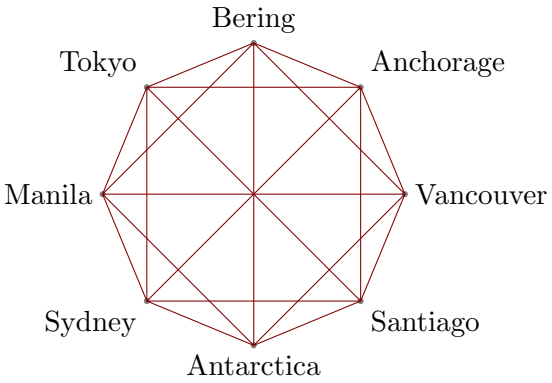


Table 3.1: Goods sold at different cities, with buying/selling price per kiloton of good. If the prices are listed as '-', it means that the particular good cannot be bought or sold there.

City	Good	Price per kT	City	Good	Price per kT
Manila	Food	50	**Vancouver**	Food	50
	Machines	-		Machines	150
	Energy	50		Energy	-
	Luxury	100		Luxury	-
	Medical	150		Medical	100
	Jewelry	200		Jewelry	50
Tokyo	Food	100	**Santiago**	Food	100
	Machines	50		Machines	50
	Energy	50		Energy	150
	Luxury	200		Luxury	100
	Medical	-		Medical	50
	Books	100		Books	200
Bering	Food	150	**Antarctica**	Food	-
	Machines	150		Machines	100
	Energy	-		Energy	-
	Luxury	-		Luxury	50
	Medical	100		Medical	200
	Isotopes	50		Isotopes	200
Anchorage	Food	200	**Sydney**	Food	50
	Machines	200		Machines	150
	Energy	50		Energy	-
	Luxury	-		Luxury	150
	Medical	-		Medical	50
	Water	100		Water	200

Example Answer:

One example of a (not-so-efficient) trade route is going back and forth between Manila and Vancouver, purchasing only Jewelry in Vancouver (filling your ship), and selling it in Manila. This answer is given in Table 3.2. The profit is then: $200 \cdot (\$200 - \$50) + 200 \cdot (\$200 - \$50) + 200 \cdot (\$200 - \$50) = \$90,000$.

Table 3.2: Example Trade Route Between Vancouver and Manila (in kT)

	Origin		1		2		3		4		5		6	
City	Manila		Vancouver		Manila		Vancouver		Manila		Vancouver		Manila	
Good	Sell	Buy	Sell	Buy	Sell	Buy	Sell	Buy	Sell	Buy	Sell	Buy	Sell	Buy
Food														
Machines														
Energy														
Luxury														
Medical														
Jewelry				200	200			200	200			200	200	
Books														
Isotopes														
Water														

Answer:

Table 3.3: **Answer:** Trade Route (in kT)

	Origin		1		2		3		4		5		6	
City	Manila												Manila	
Good	Sell	Buy	Sell	Buy	Sell	Buy	Sell	Buy	Sell	Buy	Sell	Buy	Sell	Buy
Food														
Machines														
Energy														
Luxury														
Medical														
Jewelry														
Books														
Isotopes														
Water														

Total profit: _____

4 Shopping to save [150 marks]

Background & Task

Cosmetics is a competitive business where profit margins are typically very high. Sefora Co. is a large retailer of cosmetic goods that wants to maintain its position as a market leader by further reducing their costs. The executive team has targeted sourcing as an area of high costs. *They have asked your team of operations research specialists to provide an optimal procurement plan.*

Constraints

Before rolling out the strategy for all their product lines, the executives would like to see a proof-of-concept of your methodology. Your team has decided to investigate a few items in Sefora Co.'s makeup line – foundation, concealer, blush, and eyeliner. Upon your request, the Forecasting team in your department has provided you with customer demand figures for the months of April, May and June. Their report includes numbers for the 4 product types of interest. This data is provided in Table 4.1.

Costs

Each product type has various products that can be purchased and stocked. Due to different packaging size and in-store display designs, the fixed set-up costs and variable storage costs are product specific (see Table 4.3). **Variable costs** are charged per piece of product that is stored each month. For example, if 100 units of Tinted Moisturizer are left after demand is met in April, then its storage cost for April will be \$200 (\$2.00/unit $\times$ 100 units).

Fixed Set-up Costs

The **fixed set-up cost** is paid whenever a new product is stocked for a product type's demand. As an example, if Tinted Moisturizer is used to meet the demand for foundation in all periods, then \$1,500 is paid for all 3 months. On the other hand, suppose that Tinted Moisturizer is used only for April and May and Pressed Powder is used in June. Then, the total set-up cost would be \$1,500 for using Tinted Moisturizer in April and May; and \$1,800 for changing to Pressed Powder in June.

Shipping Costs

The Vendor's Management team has also provided your team with the **unit cost** to purchase a particular product from a vendor and their available supply in Table 4.4. Each vendor has a **fixed shipping cost** for orders of any value and size and any delivery location (see Table 4.2). Note that shipping cost is only applied if a purchase is made in that time period. As an example, purchasing 1 unit of Tinted Moisturizer from ES will incur \$100 in shipping costs. Making a purchase of 100 units of Tinted Moisturizer from ES in the same period will still only cost \$100 in shipping. To be clear, purchasing some non-zero quantity of both Liquid Concealer and Tinted Moisturizer from ES in the same period will still result in the same shipping fee of \$100.

Additional Assumptions & Parameters:

Based on this information, your team can prepare a cost-minimizing procurement plan with the following additional assumptions:

1. Purchases are made at the beginning of the month and demand is consumed immediately when purchases are made. Hence, inventory holding cost is only applied to products that have not been consumed at the end of the month.

2. Demand must be met completely. Purchasing quantities in advance is allowed but at a per-unit storage cost.

3. The time required to ship the goods from the suppliers to the target retail locations will not be considered.

4. Vendor shipping policies are the same throughout all periods.

Questions:

1. Find the purchase plan over the 3 months such that the demand for each of the four product types can be met at minimum costs.

2. Your loyal relationships with the vendors have convinced some of them to provide a discount. Some vendors are willing to waive the shipping costs for a specific period when the total order volume in that period exceeds a certain threshold amount (see Table 4.2). With this new consideration and the previous assumptions, does your purchase plan change? If so, update your purchase plan.

Tables & Figures:

Table 4.1: Projected demand for each product type and the available products for stocking

		Demand (per unit)		
Product Type	Products	April	May	June
Foundation	Tinted Moisturizer, Pressed Powder	10000	12000	8000
Concealer	Liquid C, Stick C	9000	1500	9000
Blush	Powder B, Liquid B	9000	8000	7500
Eyeliner	Pencil E, Gel E, Liquid E	4500	8500	8500

Table 4.2: Shipping costs and volume-based threshold values for each vendor.
*For Part 2 - If total order volume exceeds the threshold in one month, then that month's shipping cost is waived.

Vendor	Shipping Cost	Threshold*
ES.	100	8000
GA.	75	8000
US.	100	10000
NP.	80	NA

Table 4.3: Per unit storage costs and fixed set-up costs for all products. Note that the costs remain the same in all months

Product Type	Product	Storage Cost ($ per unit)	Set-up Cost
Foundation	Tinted Moisturizer	2	1500
Foundation	Pressed Powder	2	1800
Concealer	Liquid C	1	1000
Concealer	Stick C	1	850
Blush	Powder B	1.5	1250
Blush	Liquid B	1.5	1250
Eyeliner	Pencil E	1	2000
Eyeliner	Gel E	1	2500
Eyeliner	Liquid E	0.75	2250

Table 4.4: Supply and unit costs for each product in each month

			Supply (per unit)			Unit Price ($)		
Vendor	Product Type	Product	April	May	June	April	May	June
ES.	Foundation	Tinted Moisturizer	7500	9000	6000	7.5	9	6.75
ES.	Concealer	Liquid C	8000	9500	6500	4	4.5	4
ES.	Blush	Powder B	3000	3500	2500	6.35	7.5	6
ES.	Eyeliner	Pencil E	7500	9000	6000	7	8.5	6
ES.	Eyeliner	Gel E	7500	9000	6000	7.25	9	6.5
GA.	Foundation	Tinted Moisturizer	3500	4250	3000	8	8	8
GA.	Foundation	Pressed Powder	6000	5000	5000	4.5	5.5	6.5
GA.	Concealer	Liquid C	4500	3750	3750	4.5	5.5	6
GA.	Concealer	Stick C	4000	5000	3250	4.5	5	5.5
GA.	Blush	Powder B	10000	12000	7500	6	7	7
NP.	Foundation	Tinted Moisturizer	8000	7500	6500	8	9.5	7.5
NP.	Foundation	Pressed Powder	5000	6000	4250	5.5	6.5	5.5
NP.	Blush	Powder B	10000	12000	8250	5.5	7	8
NP.	Blush	Liquid B	2500	3000	2000	5.5	7	8
NP.	Eyeliner	Gel E	5000	6000	4250	7	7.5	8
NP.	Eyeliner	Liquid E	5000	6000	4250	6.5	7.5	8
US.	Foundation	Tinted Moisturizer	2500	3000	2000	7	8	6
US.	Foundation	Pressed Powder	7500	9000	6250	5	6	5
US.	Concealer	Stick C	8500	8500	7000	5.5	6.5	5
US.	Blush	Powder B	6000	7000	5000	6	7	5
US.	Blush	Liquid B	7500	9000	6000	6	7.5	5.5
US.	Eyeliner	Pencil E	6000	4000	5000	7	8.5	6.5
US.	Eyeliner	Gel E	6000	7500	5000	7	8.5	6.5
US.	Eyeliner	Liquid E	9000	10000	7500	6.5	7	6

Answer:

Please enter your answers into Tables 4.5, 4.6, 4.7, and 4.8. If the cell is left blank, it will be assumed to be 0. For Table 4.8, answers must be 0 for NO and 1 for YES.

Table 4.5: **Answer:** Cost Categories

Cost Category	April	May	June
Order			
Shipping			
Fixed Set-up			
Storage			
Total costs per month			

Table 4.6: **Answer:** Order Quantities

		Order Quantity		
Vendor	Product	April	May	June
ES.	Tinted Moisturizer			
ES.	Liquid C			
ES.	Powder B			
ES.	Pencil E			
ES.	Gel E			
GA.	Tinted Moisturizer			
GA.	Pressed Powder			
GA.	Liquid C			
GA.	Stick C			
GA.	Powder B			
NP.	Tinted Moisturizer			
NP.	Pressed Powder			
NP.	Powder B			
NP.	Liquid B			
NP.	Gel E			
NP.	Liquid E			
US.	Tinted Moisturizer			
US.	Pressed Powder			
US.	Stick C			
US.	Powder B			
US.	Liquid B			
US.	Pencil E			
US.	Gel E			
US.	Liquid E			

Table 4.7: **Answer:** Inventory Amount

		Inventory Amount		
Product Type	Product	April	May	June
Foundation	Tinted Moisturizer			
Foundation	Pressed Powder			
Concealer	Liquid C			
Concealer	Stick C			
Blush	Powder B			
Blush	Liquid B			
Eyeliner	Pencil E			
Eyeliner	Gel E			
Eyeliner	Liquid E			

Table 4.8: **Answer:** Product Set-up

		Product Set-up		
Product Type	Product	April	May	June
Foundation	Tinted Moisturizer			
Foundation	Pressed Powder			
Concealer	Liquid C			
Concealer	Stick C			
Blush	Powder B			
Blush	Liquid B			
Eyeliner	Pencil E			
Eyeliner	Gel E			
Eyeliner	Liquid E			